1 Introduction

A patent holder has the right to prevent or exclude others from making, using, selling, offering to sell, and/or importing an invention to which he owns the rights. Thus, anyone who wants to use another entity's patent should pay royalties or the like. While this strategy's primary virtue is the provision of incentives to promote new technologies, today's high technology and its accumulation has resulted in a reverse effect of charging heavy royalties to succeeding innovators. This is called the "patent thicket problem" (Shapiro, 2000), a situation wherein the producer must pay an overlapping set of patent rights that stem from a common technology in order to commercialize a new product. Paradoxically, this requirement hinders a producer or innovator from introducing new beneficial goods or technology to the market, thereby contradicting the original purpose of the patent. This problem is particularly controversial in special key industries such as semiconductor, biotechnology, and computer software. Heller and Eisenberg (1998) refer to this problem as "the tragedy of the anti-commons", while Shapiro (2000) argues that a coordination among patent holders, such as cross-licensing and patent pools, is required to solve this problem.

In patent law, a patent pool is a consortium of at least two companies that have agreed to cross-license[1] patents relating to a particular technology, and where the entire group of patents is licensed in a bundle. This licensing policy has pro-competitive effects as well as anti-competitive effects. Tactics such as encouraging combinations among complementary technologies, reducing transition costs, evad-

[1] A cross-licensing agreement is a contract between two or more parties where each party grants rights to their intellectual property to the other parties. See Shapiro (2001).

ing an infringement litigation, and dissemination of technologies tend to promote competitiveness, while price fixing and market division may restrain competitiveness.[2]

In this paper, we restrict our focus to complementary patents for ruling out the anti-competitive effects. We will concentrate on the price-reducing patents in the Cournot market with homogeneous goods. The purpose of this paper is to examine the possibility of cooperation among patent holders. Thus, we will determine the equilibrium coalition structure and investigate whether or not the total profit of patent holders is maximized in the equilibrium. Finally, we will establish a reasonable level of bundle royalties to compensate the earlier innovator's contributions and to ensure incentives for succeeding innovators to encourage their investments.

There are two types of approaches to investigate the patent licensing policies in oligopolistic markets. First, there are non-cooperative approaches which focus on the effects of different licensing policies on the payoff of the licensor and the licensees. Kamien and Tauman (1984) compared the profitability of upfront fees and royalties, and conclude that an industry incumbent favors licensing by means of a royalty per unit of output while an outsider prefers to auction off fixed licenses. Katz and Shapiro (1985) showed that major innovations would not be licensed under the upfront fee policy. Kamien and Tauman (1986), and Kamien et al. (1992) discussed the licensing by means of an auction. Meanwhile, Muto (1993) compared these licensing policies in the Bertrand duopoly with differentiated commodities. These papers dealt with the licensing contracts between an innovator who can-

[2] US.DOI and FIC, "Antitrust Guidelines for the Licensing of Intellectual Property," Apr. 6, 1995, p28.

not produce goods by himself and several producers who compete in the market. Secondly, there are cooperative approaches such as the stable profit sharing in a patent licensing game in Watanabe and Muto (2004) and the Shapley value of a patent licensing game in Tauman and Watanabe (2006). By assuming a maxmin or minmax strategy for residual players, these papers pin down the possible externalities among players. Thus the Shapley value for the cooperative TU game plays an important role as an allocation rule between licensor and licensees.

The differences between this paper and previous studies are as follows. First, we assume that some firms competing in the market are also patent holders, whereas the previous studies assume the innovator is outside the market. Second, we derive the worth of coalitions with the partition function form game which explicitly considers the externalities among coalitions. Third, we assume that the patent pools are formed in the process of the alternating-offers bargaining game, which was introduced by Rubinstein (1982), and then extended to n players by Selten (1981) and Chattergee et al. (1993). While this bargaining procedure was already applied to the symmetric oligopolistic market by Bloch (1995), our model considers the case where firms are not necessarily patent holders, thus the number of competitors and the number of patent holders can be different each other. From this assumption, we found that a grand coalition can be formed in the equilibrium especially when the number of patent holders is relatively smaller the number of whole competitors. Additionally, based on the equilibrium of our model, we investigate which equilbrium coalition structure can maximize the ex-post total profits of patent holders. Finally, we examine whether or not a patent pool can solve the patent thicket problem as Shapiro (2000) suggested, and establish a lower bound for bundle royalties, which is the main result of this paper.

The outline of this paper is as follows. In Section 2, we introduce the basic concept of the partition function form game to facilitate better understanding of our model. In Section 3, we describe our model and derive the worth of coalitions with the partition function form game. In Section 4, we find the Markov-perfect equilibrium (equilibria) and corresponding equilibrium coalition structures in the symmetric case, and examine whether total profit of patent holders is maximized in that equilibrium. In section 5, we compare our result with the patent thicket problem, and finally, we present our conclusion in Section 6.

2 Preliminaries

Let $N = \{1, \cdots, n\}$ be the set of players. A partition of N into coalitions $Q = \{S_1, S_2, \cdots, S_k\}$ is a coalition structure of N, if $S_i \cap S_j = \varnothing, \forall\ i \neq j,$ and $\bigcup_{i=1}^{k} S_i = N$. Let $\wp(N)$ be the set of all partitions of N, and a typical element of $\wp(N)$ is denoted by Q. A pair (S, Q) which consists of a coalition S and a partition Q of N to which S belongs is called an embedded coalition, i.e.

$$E(N) = \left\{(S, Q) \in 2^N \times \wp(N) \mid S \in Q\right\}$$

Thus a coalition structure is an element of $E(N)$. The induced partition on T is denoted by $(T, Q(T))$ with

$$Q(T) = \{T \cap S \mid S \in Q \text{ and } T \cap S \neq \varnothing\}$$

A pair (T, Q) which consists of a coalition T and a partition Q of N is called an induced coalition, i.e.

$$I(N) = \left\{(T, Q) \in 2^N \times \wp(N) \mid T \subset N\right\}$$

Definition 1 A mapping

$$w : I(N) \to \mathbb{R}$$

that assigns a real valiue, $w(T, Q)$ to each induced coalition (T, Q) is called a partition function, and the ordered pair (N, w) is called a partition function form game. We compute $w(T, Q) = \sum_i w(Q(T)_i, Q)$, where $Q(T)_i \in Q(T)$.

The partition which consists of singleton coalitions, $Q = \{\{1\}, \cdots, \{n\}\}$, is denoted $[N]$,whereas the partition which consists of a grand coalition is denoted by $\{N\}$. For any subset $S \subseteq N$, let $[S]$ denote the typical partition which consists of the singleton elements of S, i.e.$[S] = \{\{j\} \mid j \in S\}$.

3 The model

3.1 The Cournot market with several complementary patents

We consider a Cournot market with a linear demand curve $P = a - Q$ in which firms produce homogeneous products at a fixed marginal cost, c. The market clears at the price $p = \max(a - \sum_{i \in N} q_i, 0)$, where $a \in (c, \infty)$ is a constant. Let $N = \{1, \cdots, n\}$ be the set of firms in this market, where $2 \leq n < \infty$, and let $M = \{1, \cdots, m\}$ $(M \subset N)$ be the firms with patents which reduces the marginal cost by the amount of $\varepsilon_1, \cdots, \varepsilon_m$ respectively. In this paper, we assume firms cannot form a cartel to produce together (by law), but they can form a patent pool to reduce their marginal cost together. Assuming that all patents are complementary to each other, the cost reduction of any combination of patents is the sum of each patent's cost reduction. More precisely, using k patents simultaneously, a firm can reduce their marginal cost by the amount of $\sum_{i=1}^{k} \varepsilon_i$, where $k = 1, \cdots m$. If a patent pool wants to perfectly exclude others from using their own patents, they might determine bundle royalties as $\sum_{i=1}^{k} \varepsilon_i$.[3] Thus outsider of a patent pool has no incentive to use that pool's patents. As a result, after patent pools are formed, the marginal cost function of firm i belonging to the pool S_j is $c - \sum_{i \in S_j} \varepsilon_i$ for $i \in S_j \subset M$ and c for $i \in N \backslash M$.

The formation of patent pools in this market is analized as a two-stage non-cooperative game. In the first stage, patent holders form a patent pool to reduce their marginal cost. And in the second stage, each firm competes in the Cournot market with or without cost advanteges according to the set of patents they can use.

3.2 The worth of a coalition in the patent pool game

In this part, we restrict our focus to the second stage of the game, and derive a partition function given a coalition structure. After all patent pools are formed, every firm maximizes the individual profit competing in the market. Due to the externalities among players, each firm's profit depends not only on his own behavior but also on other firms' behaviors. Thus the structure of coalitions should be internalized in the worth of each coalition. As introduced in the previous section, a partition function is the concept to describe this situation, defined on the pair of a coalition and a particular coalition structure. To define a partition function for this game, first, we define a partition of N as follows. Since the cooperation is only possible among the patent holders, the possible coalition structure can be written as

[3] If the cost reduction is relatively small and the innovator will act as a monopolist with no threat of entry, there is no need for licensing. Therefore $r_i = \varepsilon_i$ for $i = 1, \cdots, m$. More literature on the best way to license cost-reducing innovations, see Kamien (1992).

$$Q = \{S_1, S_2, \cdots, S_k, [N \backslash M]\} \quad \text{where}$$

i) $1 \leq k \leq m, \quad S_i \cap S_j = \varnothing \ \forall \ i \neq j$ and $\bigcup_{i=1}^{k} S_i = M$,

ii) $[N \backslash M] = \{\{j\} \mid j \in N \backslash M\}$

Let $\varepsilon_i = 0$ for $i \in N \backslash M$. Then, assuming that all patents are complement to each other, the marginal cost of each firm given a coalition structure Q is

$$C(i) = c - \sum_{i \in S_j} \varepsilon_i, \quad if \ \ i \in N.$$

,where $S_j \in Q$, for $j = 1, \cdots, k$.

There would be R&D costs to get a patent. Assuming decreasing returns to scale, R&D cost can be expressed as follows.

$$I(\varepsilon_i) = \frac{\gamma}{2} \varepsilon_i^2 \quad \text{, where } \gamma > 0$$

However, this R&D cost is usually considered as a lumpsum cost which does not affect firm's strategy when they choose their production level in the Cournot market. Thus, in this paper, we limit our analysis to the marginal cost.

After all patent pools are formed, firms behave as competitors in the Cournot market and maximize the individual profits. The Cournot-Nash equilibrium given the coalition structure $Q = \{S_1, S_2, \cdots, S_k, [N \backslash M]\} \in \wp(N)$ is the vector $(q_1^*, q_2^*, \cdots, q_n^*)$ which satisfies

$$\pi_{S_k,Q}^i \left(q_i^*, q_{-i}^*\right) \geq \pi_{S_k,Q}^i \left(q_i, q_{-i}^*\right) \ , \quad \text{for all } i \in N \ .$$

Suppose that $\varepsilon_j = 0$, for $j \in N \backslash M$. Then the profit function of firm i is

$$\pi_i = (a - c - \sum_{i\in N} q_i + \sum_{i\in S_k} \varepsilon_i)q_i, \text{ for } i = 1, \cdots, n$$

Thus the first order condition of firm's profit maximization problem is

$$\frac{\partial \pi_i}{\partial q_i} = (a - c - \sum_{i\in N} q_i + \sum_{i\in S_k} \varepsilon_i) - q_i = 0, \text{ for } i = 1, \cdots, n$$

Proposition 1. In the patent pool game $(N, M, \wp, w_{S,Q})$, the profit of firm $j \in S_i \in Q$ given the coalition structure $Q = \{S_1, \cdots, S_k, [N\backslash M]\}$ is

$$\pi_j^* = \left[\frac{a-c+(n-|S_i|+1)\sum_{j\in S_i}\varepsilon_j - \sum_{m\neq i}|S_m|\sum_{j'\in S_m}\varepsilon_{j'}}{n+1}\right]^2$$

,where $|S_i|$ is the number of members in S_i.

Proof. Given a coalition structure $Q = \{S_1, \cdots, S_k, [N\backslash M]\}$, the first order necessary condition and the second order sufficient condition yield firm j's best reponse function. Since the profit $\pi_j (j \in N)$ is strictly concave with respect to q_j, the second order condition is always satisfied. Thus it suffices to check the first order condition. The following equation shows each firm's reaction function.

$$\frac{\partial \pi_i}{\partial q_i} = (a - c - \sum_{i\in N} q_i + \sum_{i\in S_k} \varepsilon_i) - q_i = 0, \text{ for } i = 1, \cdots, n$$

Without loss of generality, we rearrange firms according to the order of thier coalition S_i. Let $\sigma : N \Rightarrow N$ be a bijection such that if $g \leq h$, then $\sigma(j) \leq \sigma(l)$, where $j \in S_g, l \in S_h$. The Nash equilibrium production level is the solution of following linear system.[4]

[4]The unique existence of the Nash equilibrium corresponding to any coalition structures is guaranteed by assumptions made on market competition.

$$\begin{pmatrix} 2 & 1 & 1 & \cdots & 1 & 1 & 1 \\ 1 & 2 & 1 & \cdots & 1 & 1 & 1 \\ 1 & 1 & 2 & \cdots & 1 & 1 & 1 \\ 1 & 1 & 1 & \ddots & 1 & 1 & 1 \\ 1 & 1 & 1 & \cdots & 2 & 1 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 2 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ \vdots \\ q_{n-2} \\ q_{n-1} \\ q_n \end{pmatrix} = \begin{pmatrix} a-c+\sum_{i \in S_1} \varepsilon_i \\ \vdots \\ a-c+\sum_{i \in S_2} \varepsilon \\ \vdots \\ a-c+\sum_{i \in S_k} \varepsilon_i \\ \vdots \\ a-c \end{pmatrix}$$

The dimension of each matrix is $n \times n, n \times 1, n \times 1$, respectively. The first matrix in the equation always have its inverse matrix as follows.

$$\begin{pmatrix} 2 & 1 & 1 & \cdots & 1 & 1 & 1 \\ 1 & 2 & 1 & \cdots & 1 & 1 & 1 \\ 1 & 1 & 2 & \cdots & 1 & 1 & 1 \\ 1 & 1 & 1 & \ddots & 1 & 1 & 1 \\ 1 & 1 & 1 & \cdots & 2 & 1 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 2 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 1 & 2 \end{pmatrix}^{-1} = \frac{1}{n+1} \begin{pmatrix} n & -1 & -1 & \cdots & -1 & -1 & -1 \\ -1 & n & -1 & \cdots & -1 & -1 & -1 \\ -1 & -1 & n & \cdots & -1 & -1 & -1 \\ -1 & -1 & -1 & \ddots & -1 & -1 & -1 \\ -1 & -1 & -1 & \cdots & n & -1 & -1 \\ -1 & -1 & -1 & \cdots & -1 & n & -1 \\ -1 & -1 & -1 & \cdots & -1 & -1 & n \end{pmatrix}$$

The Nash equilibrium production level is as follows.

$$\begin{pmatrix} q_1^* \\ q_2^* \\ q_3^* \\ \vdots \\ q_{n-2}^* \\ q_{n-1}^* \\ q_n^* \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 & \cdots & 1 & 1 & 1 \\ 1 & 2 & 1 & \cdots & 1 & 1 & 1 \\ 1 & 1 & 2 & \cdots & 1 & 1 & 1 \\ 1 & 1 & 1 & \ddots & 1 & 1 & 1 \\ 1 & 1 & 1 & \cdots & 2 & 1 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 2 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} a-c+\sum_{i \in S_1} \varepsilon_i \\ \vdots \\ a-c+\sum_{i \in S_2} \varepsilon \\ \vdots \\ a-c+\sum_{i \in S_k} \varepsilon_i \\ \vdots \\ a-c \end{pmatrix}$$

$$= \frac{1}{n+1}\begin{pmatrix} n & -1 & -1 & \cdots & -1 & -1 & -1 \\ -1 & n & -1 & \cdots & -1 & -1 & -1 \\ -1 & -1 & n & \cdots & -1 & -1 & -1 \\ -1 & -1 & -1 & \ddots & -1 & -1 & -1 \\ -1 & -1 & -1 & \cdots & n & -1 & -1 \\ -1 & -1 & -1 & \cdots & -1 & n & -1 \\ -1 & -1 & -1 & \cdots & -1 & -1 & n \end{pmatrix}\begin{pmatrix} a-c+\sum_{i\in S_1}\varepsilon_i \\ \vdots \\ a-c+\sum_{i\in S_2}\varepsilon_i \\ \vdots \\ a-c+\sum_{i\in S_k}\varepsilon_i \\ \vdots \\ a-c \end{pmatrix}$$

$$= \frac{1}{n+1}\begin{pmatrix} a-c+(n-|S_1|+1)\sum_{i\in S_1}\varepsilon_i - \sum_{m\neq 1}|S_m|\sum_{j\in S_m}\varepsilon_j \\ \vdots \\ a-c+(n-|S_2|+1)\sum_{i\in S_2}\varepsilon_i - \sum_{m\neq 2}|S_m|\sum_{j\in S_m}\varepsilon_j \\ \vdots \\ a-c+(n-|S_k|+1)\sum_{i\in S_k}\varepsilon_i - \sum_{m\neq k}|S_m|\sum_{j\in S_m}\varepsilon_j \\ \vdots \\ a-c \end{pmatrix}$$

Thus, the profit of firm $j \in S_i \in Q$ given the coalition structure $Q = \{S_1, \cdots, S_k, [N\backslash M]\}$ is

$$\pi_j^* = \left[\frac{a-c+(n-|S_i|+1)\sum_{j\in S_i}\varepsilon_j - \sum_{m\neq i}|S_m|\sum_{j'\in S_m}\varepsilon_{j'}}{n+1}\right]^2$$

,where $|S_i|$ is the number of members in S_i. Q.E.D.

Assuming that patent holders cannot form a cartel in the second stage, the worth of a coalition is defined as the sum of its member's profits.

Definition 2 In the patent pool game $(N, M, \wp, w_{T,Q})$, the worth of a coalition T given a coalition structure $Q = \{S_1, \cdots, S_k, [N\backslash M]\} \in \wp$ is

$$w_{T,Q} = \sum_{Q(T)_l \in Q(T)} \sum_{j \in Q(T)_l} \left[\frac{a-c+(n-|S_i|+1)\sum_{j\in S_i}\varepsilon_j - \sum_{m\neq i}|S_m|\sum_{k\in S_m}\varepsilon_k}{n+1} \right]^2$$

Thus, the worth of a coalition S_i given a coalition structure $Q = \{S_1, \cdots, S_k, [N\backslash M]\} \in \wp$ is

$$\begin{aligned} w_{S_i,Q} &= \sum_{j\in S_i} \left[\frac{a-c+(n-|S_i|+1)\sum_{j\in S_i}\varepsilon_j - \sum_{m\neq i}|S_m|\sum_{k\in S_m}\varepsilon_k}{n+1} \right]^2 \\ &= |S_i| \cdot \left[\frac{a-c+(n-|S_i|+1)\sum_{j\in S_i}\varepsilon_j - \sum_{m\neq i}|S_m|\sum_{k\in S_m}\varepsilon_k}{n+1} \right] \end{aligned}$$

Example 1. Suppose the set of firms and patent holders in the Cournot market are $N = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $M = \{1, 2, 3, 4, 5\}$ respectively, and the coalition structure is given as $Q = \{\{1, 2, 3\}, \{4, 5\}, 6, 7, 8\}$. Each firm i, $(i = 1, 2, 3, 4, 5)$ has his own patent which reduces the marginal cost by the amount of ε_i. In this situation, the Nash equilibrium production level and the profit of each firm can be derived as follows.

$$\begin{pmatrix} q_1^* \\ q_2^* \\ q_3^* \\ q_4^* \\ q_5^* \\ q_6^* \\ q_7^* \\ q_8^* \end{pmatrix} = \frac{1}{9} \begin{pmatrix} a-c+6(\varepsilon_1+\varepsilon_2+\varepsilon_3)-2(\varepsilon_4+\varepsilon_5) \\ a-c+6(\varepsilon_1+\varepsilon_2+\varepsilon_3)-2(\varepsilon_4+\varepsilon_5) \\ a-c+6(\varepsilon_1+\varepsilon_2+\varepsilon_3)-2(\varepsilon_4+\varepsilon_5) \\ a-c+7(\varepsilon_4+\varepsilon_5)-3(\varepsilon_1+\varepsilon_2+\varepsilon_3) \\ a-c+7(\varepsilon_4+\varepsilon_5)-3(\varepsilon_1+\varepsilon_2+\varepsilon_3) \\ a-c-3(\varepsilon_1+\varepsilon_2+\varepsilon_3)-2(\varepsilon_4+\varepsilon_5) \\ a-c-3(\varepsilon_1+\varepsilon_2+\varepsilon_3)-2(\varepsilon_4+\varepsilon_5) \\ a-c-3(\varepsilon_1+\varepsilon_2+\varepsilon_3)-2(\varepsilon_4+\varepsilon_5) \end{pmatrix}$$

$$\begin{pmatrix} \pi_1^* \\ \pi_2^* \\ \pi_3^* \\ \pi_4^* \\ \pi_5^* \\ \pi_6^* \\ \pi_7^* \\ \pi_8^* \end{pmatrix} = \begin{pmatrix} \frac{1}{81}\left(a-c+6(\varepsilon_1+\varepsilon_2+\varepsilon_3)-2(\varepsilon_4+\varepsilon_5)\right)^2 \\ \frac{1}{81}\left(a-c+6(\varepsilon_1+\varepsilon_2+\varepsilon_3)-2(\varepsilon_4+\varepsilon_5)\right)^2 \\ \frac{1}{81}\left(a-c+6(\varepsilon_1+\varepsilon_2+\varepsilon_3)-2(\varepsilon_4+\varepsilon_5)\right)^2 \\ \frac{1}{81}(a-c+7(\varepsilon_4+\varepsilon_5)-3(\varepsilon_1+\varepsilon_2+\varepsilon_3))^2 \\ \frac{1}{81}(a-c+7(\varepsilon_4+\varepsilon_5)-3(\varepsilon_1+\varepsilon_2+\varepsilon_3))^2 \\ \frac{1}{81}\left(a-c-3(\varepsilon_1+\varepsilon_2+\varepsilon_3)-2(\varepsilon_4+\varepsilon_5)\right)^2 \\ \frac{1}{81}\left(a-c-3(\varepsilon_1+\varepsilon_2+\varepsilon_3)-2(\varepsilon_4+\varepsilon_5)\right)^2 \\ \frac{1}{81}\left(a-c-3(\varepsilon_1+\varepsilon_2+\varepsilon_3)-2(\varepsilon_4+\varepsilon_5)\right)^2 \end{pmatrix}$$

Thus the worth of the embedded coalitions is as follows.

$$\begin{pmatrix} w_{\{123\},Q} \\ w_{\{45\},Q} \\ w_{\{6\},Q} \\ w_{\{7\},Q} \\ w_{\{8\},Q} \end{pmatrix} = \begin{pmatrix} \frac{1}{27}\left(a-c+6(\varepsilon_1+\varepsilon_2+\varepsilon_3)-2(\varepsilon_4+\varepsilon_5)\right)^2 \\ \frac{2}{81}(a-c+7(\varepsilon_4+\varepsilon_5)-3(\varepsilon_1+\varepsilon_2+\varepsilon_3))^2 \\ \frac{1}{81}\left(a-c-3(\varepsilon_1+\varepsilon_2+\varepsilon_3)-2(\varepsilon_4+\varepsilon_5)\right)^2 \\ \frac{1}{81}\left(a-c-3(\varepsilon_1+\varepsilon_2+\varepsilon_3)-2(\varepsilon_4+\varepsilon_5)\right)^2 \\ \frac{1}{81}\left(a-c-3(\varepsilon_1+\varepsilon_2+\varepsilon_3)-2(\varepsilon_4+\varepsilon_5)\right)^2 \end{pmatrix}$$

The worth of the induced coalition $T = \{1, 2, 4\}$ is

$$\begin{aligned} w_{\{12\}\cup\{4\},Q} = {} & \frac{2}{81}\left(a-c+6(\varepsilon_1+\varepsilon_2+\varepsilon_3)-2(\varepsilon_4+\varepsilon_5)\right)^2 \\ & +\frac{1}{81}(a-c+7(\varepsilon_4+\varepsilon_5)-3(\varepsilon_1+\varepsilon_2+\varepsilon_3))^2 \end{aligned}$$

4 The sequential coalition formation in the patent pool game with externality

In this section, we describe the formation procedure of patent pools, which occurs in the first stage of the game. To rule out the effect of allocation rules, we restrict our focus to the symmetric case, where all patents reduce the cost at the same amount. We assume that patent holders negotiate on forming a patent pool in the spirit of the alternating-offers bargaining game. This bargaining game was introduced by Rubinstein (1982), extended to n players by Selten (1981) and Chattergee et al. (1993), and then applied to the symmetric oligopolistic market by Bloch (1995). One explicit difference of our model from Bloch's is that in our model firms are not necessarily patent holders, thus the number of competitors and the number of patent holders can be different from each other.

At the beginning of the game, an order of being a proposer is given as $\omega = (\omega_1, \cdots, \omega_m)$, where $\omega : M \to M$. Only cooperations are possible among patent holders becuase forming a cartel is prohibited (by law). The procedure of the bargaining game is as follows. One firm chosen as the initiator proposes an association to some patent holders in the market. Then, all prospective members respond to that offer in turn. If all of them agree to cooperate, the proposed patent pool will be formed and the residual players will start the next round for another patent pool. Otherwise, the proposed patent pool fails to form and the second proposer will get a chance to propose the alternative cooperation. To rule out the effect of arbitrary termination rules, assume the horizon of the game is infinite. Firms do not discount payoffs, but in the case of an infinite play of the game, all firms receive a payoff of zero (Selten, 1981).

Formally, a list of all actions taken by the players from period 1 to period t-1 is a history at period t. A strategy σ_i for firm i is a choice of action at t contingent to each history. An action can either be the choice of cooperation or a response to a proposal (Yes or No). Since the past has a direct influence on current opportunities by changing a coalition structure, the equilibrium concept in this model is a Markov-perfect equilibrium (MPE): (a) every firm's strategy is a Markov strategy which depends only on the ongoing proposal and the cooperation formed in the previous periods; b) σ_i is the best response to σ_{-i} after every history at which i moves. A strategy for a player involves how one makes a proposal whenever it is his turn to do so and whether one accepts or rejects proposals at every stage where he is required to respond.

An outcome of this bargaining game is a partition of the set of patent holders into disjoint cooperations called "patent pools." Moreover, payoffs to the firms are determined as the profits obtained in the second stage of the game. A perfect equilibrium is a profile of strategies where there is no history that deviating from one's strategy from the prescribed strategy leaves one better off. A coalition structure Q generated by a Markov-perfect equilibrium of the game is called an equilibrium coalition structure.

In the symmetric case where the cost reduction amount of all patents are equal, the equilibrium structure of the alternating-offers bargaining game coincides with the choice-of-size game up to a permutation of the firms. This result follows from the results of Bloch (1995,1996). In this game, a firm's strategy is either be announcing an integer as a pool size or responding to a proposal (Yes or No). The initiator announces an integer a_1 as the size of the pool to be formed, and the $(i-1)$ prospective pool members respond to that proposal. If all of them agree, then the game goes to the next round. Then, the next proposer whose ranking is highest among the remaining firms chooses an integer a_2. The game proceeds until the coalition size $a_1, a_2, \cdots, a_R$ satisfy $\sum_r a_r = m$. From the above assumptions, we will find the equilibrium coalition structure in the patent pool game.

4.1 The equilibrium coalition structure in the symmetric case

Assuming all patents reduce the marginal cost at the same amount, say ε, then by the definition 2, the worth of a coalition S_i (where $i = 1, \cdots, k$) given the coalitoin structure $Q = \{S_1, \cdots, S_k, [N \backslash M]\}$ can be written as

$$w_{S_i,Q} = \left[\tfrac{a-c+(n-|S_i|+1)|S_i|\varepsilon - \sum_{m\neq i}|S_m|^2\varepsilon}{n+1}\right]^2 \cdot |S_i|$$

In this symmetric situation, the ex-post payoff of each member of the patent pool S_i is determined as

$$\Phi_{S_i,Q} = \left[\tfrac{a-c+(n-|S_i|+1)|S_i|\varepsilon - \sum_{m\neq i}|S_m|^2\varepsilon}{n+1}\right]^2$$

The fact that the worth of coalition S_i and the individual payoff $\Phi_{S_i,Q}$ depend on how $N \backslash S_i$ form their own patent pools shows externalities among patent holders. Our question is that whether we can find any deterministic equilibrium coalition structure in this game, in which payoffs and coalition structures interwork through the dynamic process. In order to make the rule for negotiation, we apply the alternating-offers bargaining game to our model.

Proposition 2. Let n be the total number of firms and m be the number of patent holders in the Cournot market. Suppose all patents are symmetric and complementary to each other. Then the Markov-perfect equilibrium coalition structure in the patent pool game $(N, M, \wp, w_{T,Q})$ is $Q^* = \{M, [N \backslash M]\}$ for $m < \frac{n+1}{2}$ and $Q^* = \{S_1, S_2, [N \backslash M]\}$ for $m > \frac{n+1}{2}$, where $[N \backslash M] = \{\{j\} \mid j \in N \backslash M\}$, $S_2 = M \backslash S_1$, and $S_1 = \left\{S_1 \subset M \mid |S_1| = \left[\frac{n+1+2m}{4}\right]\right\}$ if $0 \leq \alpha \leq 0.5$, and $S_1 = \left\{S_1 \subset M \mid |S_1| = \left[\frac{n+1+2m}{4}\right] + 1\right\}$ if $0.5 \leq \alpha \leq 1$.

Proof. Since in the games with perfect information, a monotonic transformation does not affect the equilibrium. Thus consider $q_i^*(S_k, Q) = \sqrt{\Phi^*_{S_k,Q}(i)}$, and examine the superadditivity among two coalitions when other coalitions don't change their cooperation. The incentive of members in coalition S_l to merge with coalition S_m is

$$q_i^{*'} - q_i^* = \sqrt{\Phi^*_{S_l \cup S_m, Q'}(i)} - \sqrt{\Phi^*_{S_l,Q}(i)} = \frac{|S_m|}{n+1}(n + 1 - 2\,|S_l|)\varepsilon.$$

The incentive for the members of S_l to merge with S_m is positive until $|S_l| < \frac{n+1}{2}$, thus in the Markov-perfect equilibrium, two asymmetric patent pools would be formed.

The initiator also knows this result ; if he chooses the size of the pool to be a_1, then the remaining patent holders will form one another pool whose size is $m - a_1$. Thus, the payoff of initiator in S_1 is to be

$$\Phi^*_{S_1,Q} = \left[\frac{a-c+(n-a_1+1)a_1\varepsilon-(m-a_1)^2\varepsilon}{n+1}\right]^2$$

Again, by the monotonic transformation, it suffices to find the integer $a_1$which maximizes the value of q_1^*. However, since q_1^* is strictly concave with respect to a_1, the second order sufficient condition always holds. The first order necessary condition is

$$\frac{\partial q_1^*}{\partial a_1} = \frac{1}{n+1}(n + 2m + 1 - 4a_1)\varepsilon = 0$$

Thus the optimal size of the first coalition is $a_1^* = \min\left\{\frac{n+2m+1}{4}, m\right\}$. To make a_1^* be integer, let us consider $\frac{n+2m+1}{4} = k + \alpha$ (k is an integer and $0 \le \alpha < 1$) and compare $q_1^*(k)$ and $q_1^*(k + 1)$.

$$q_1^*(k) - q_1^*(k+1) = \frac{a-c+(n-k+1)k\varepsilon-(m-k)^2\varepsilon}{n+1} - \frac{a-c+(n-k)(k+1)\varepsilon-(m-k-1)^2\varepsilon}{n+1}$$
$$= \frac{\varepsilon}{n+1}\left(4k+1-2m-n\right)$$

We have 4 cases as follows.

i) $0 \leq \alpha < \frac{1}{4} \Rightarrow 4k \leq n+2m+1 < 4k+1 \Rightarrow 4k+1-2m-n > 0.$

ii) $\frac{1}{4} \leq \alpha < \frac{1}{2} \Rightarrow 4k+1 \leq n+2m+1 < 4k+2 \Rightarrow 4k+1-2m-n > 0.$

iii) $\frac{1}{2} \leq \alpha < \frac{3}{4} \Rightarrow 4k+2 \leq n+2m+1 < 4k+3 \Rightarrow 4k+1-2m-n \leq 0.$

iv) $\frac{3}{4} \leq \alpha < 1 \Rightarrow 4k+3 \leq n+2m+1 < 4k+4 \Rightarrow 4k+1-2m-n \leq 0.$

Thus, if $\alpha = 0$ or 0.25, then $a_1^* = \left[\frac{n+2m+1}{4}\right]$, and if $\alpha = 0.75$, then $a_1^* = \left[\frac{n+2m+1}{4}\right]+1$. Finally, if $\alpha = 0.5$, then $a_1^* = \left[\frac{n+2m+1}{4}\right]$ or $\left[\frac{n+2m+1}{4}\right]+1$. To sum up, the Markov-perfect equilibrium structure is $Q^* = \{M, [N\backslash M]\}$ for $m < \frac{n+1}{2}$, $Q^* = \{S_1, S_2, [N\backslash M]\}$ for $m > \frac{n+1}{2}$, where $[N\backslash M] = \{\{j\} \mid j \in N\backslash M\}$, $S_2 = M\backslash S_1$ and $|S_1| = \left[\frac{n+1+2m}{4}\right]$ if $\alpha = 0$ or 0.25 or 0.5 and $|S_1| = \left[\frac{n+1+2m}{4}\right]+1$ if $\alpha = 0.5$ or 0.75. Q.E.D.

Example 2. Suppose there are 12 firms competing in the Cournot market with homogeneous goods. Among them, 10 firms have cost reducing patents which are complementary to each other, and let $M = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. The order to be a proposal is given as $\omega(i) = i$, where $i = 1, \cdots, 10$. We assume that all patents reduce by the same amount, i.e., all patents are symmetric.

In this situation firm 1 proposes $|S_1|^* - 1 = \left[\frac{12+20+1}{4}\right] - 1 = 7$ numbers of patent holders to join their patent pool . Suppose that firm $2, 3, 5, 6$ $8, 9, 10$ are offered this proposal. Since joinig this patent pool is most profitable to prospective members, all of them will accept this offer. After the first patent pool has been formed, firm 4 tries to propose another patent pool. In this round, firm 4 asks all residual players to join the second patent pool, and all prospective members agree to cooperation. Thus the equilibrium coalition structure in this game is $Q^* = \{\{1, 2, 3, 5, 6, 8, 9, 10\}, \{4, 7\}, 11, 12\}$.

4.2 The total profit maximization

In this section, we examine whether or not the equilibrium coalition structure maximizes total profit of patent holders from the ex-post point of view. Since the individual payoff is the sole criterion for the firm's decision making, we cannot assure that the total profit of patent holders are also maximized in that equilibrium. If there is another coalition structure with larger total profit, all patent holders could be better off with proper side-payments. We found that if $m < \frac{n+1}{2}$, the equilibrium coalition structure also maximizes the total profit of patent holders, while it is not true in the case of $m > \frac{n+1}{2}$. In other words, a grand coalition generated in the equilibrium always results the largest total profit, while two asymmetric patent pools might not.

Proposition 3 In the patent pool game, the total profit of patent holders is maximized in the equilibrium, if $m < \frac{n+1}{2}$.

Proof By the proposition 2, the Markov-perfect equilibrium coalition is a grand coalition among patent holders. The total profit of patent holders in this equilibrium is

$$\left(\frac{a-c+(n-m+1)m\varepsilon}{n+1}\right)^2 \times m$$

Now assume that another coalition structure is formed in the equilbirium. As we have seen in the previous section, the equilibrium coalition structure consists of at most two coalitions. Suppose that $Q = \{S_1, M\backslash S_1, [N\backslash M]\}$, where $|S_1| = k$. Then, the total payoff of patent holders is

$$\left(\frac{a-c+(n-k+1)k\varepsilon-(m-k)^2\varepsilon}{n+1}\right)^2 \cdot k + \left(\frac{a-c+(n-m+k+1)(m-k)\varepsilon-k^2\varepsilon}{n+1}\right) \cdot (m-k)$$

However, since $k \leq m < \frac{n+1}{2}$,

$$\begin{aligned} a-c+(n-k+1)k\varepsilon-(m-k)^2\varepsilon &= a-c+(n-m+1+(m-k))(m-m+k)\varepsilon-(m-k)^2\varepsilon \\ &= a-c+(n-m+1)m\varepsilon-(m-k)(n+1-2k) \\ &\leq a-c+(n-m+1)m\varepsilon \end{aligned}$$

And

$$\begin{aligned} &a-c+(n-m+k+1)(m-k)\varepsilon-k^2\varepsilon \\ &= a-c+(n-m+1)m\varepsilon-(n+1-2m)\varepsilon-2k^2\varepsilon \\ &\leq a-c+(n-m+1)m\varepsilon \end{aligned}$$

Thus for all $1 \leq k \leq m$,

$$\left(\frac{a-c+(n-m+1)m\varepsilon}{n+1}\right)^2 \times m \geq$$
$$\left(\frac{a-c+(n-k+1)k\varepsilon-(m-k)^2\varepsilon}{n+1}\right)^2 \cdot k + \left(\frac{a-c+(n-m+k+1)(m-k)\varepsilon-k^2\varepsilon}{n+1}\right) \cdot (m-k)$$

Q.E.D.

If $m > \frac{n+1}{2}$, the proposition 2 says two coalitions are formed in the equilibrium, but we cannot assure that the coalition structure maximizes the total profit of patent holders. Consider the following example. We can see that forming two asymmetric coalitions is better for members of the first patent pool. However, Markov-perfect equilibrium coalition structure doesn't maximize the total profit of patent holders compared to forming a grand coalition. For the conveninence, let us denote forming two asymmetric coalitions according to the Markov-perfect equilibirum to be 'state 1', and forming a grand coalition to be 'state 2'.

The following table shows relationships between 4 parameters (a, c, m, ε) and firm's profit, where $a - c = 20$, $n = 20$. The joint effect of ε and m on the individual profit is described in the table 1.1, and the joint effect of ε and m on the total profit of patent holders is described in the table 1.2.

$m\backslash\varepsilon$	1	0.5	0.1
11	0	0	0
12	0.6	0.2	0.01
13	1.7	0.3	0.04
14	3.3	1.0	0.08
15	4.7	1.5	0.12
16	7.3	2.2	0.20
17	9.4	2.8	0.27
18	11.2	3.4	0.34
19	12.4	3.8	0.42
20	12.8	4.2	0.49

<table 1.1>

$\triangle\ \Phi = \Phi_1^1 - \Phi_1^2$

$m\backslash\varepsilon$	1	0.5	0.1
11	0	0	0
12	-15.9	-8.5	-1.8
13	10.1	-17.8	-1.5
14	9.9	-5.1	-2.6
15	68.8	10.1	-3.5
16	88.4	13.3	-2.3
17	179.9	37.4	-0.9
18	210.3	45.5	-0.6
19	325.9	52.4	1.3
20	350.3	84.2	2.3

<table 1.2>

$\triangle\ w = w_1^1 - w_1^2$

Let $\triangle \, \Phi = \Phi_1^1 - \Phi_1^2$, where Φ_1^1 is the profit of a firm belonging to the dominant patent pool in the 'state 1' and Φ_1^2 is the payoff of each firm in the 'state 2'. And let $\triangle \, w = w_1^1 - w_1^2$, where w_1^1 is the total profit of the patent holders in the 'state 1', and w_1^2 is the total payoffs in the 'state 2'. From the table 1.1, we can see that regardless of the amount of ε, the difference of each firm's profit between 'state 1' and 'state 2' increases as the number of patent holders goes up to n. Thus, in the perspective of each individual firm, forming a dominant coalition rather than a grand coalition is better off.

However, from the table 1.2, we can see that the sequential coalition formation does not gaurantee the ex-post total profit maximization of all patent holders. As the amount of cost reduction ε decreases, total profit in the Markov-perfect equilibrium coalition structure tends to be lower than that of a grand coalition. For example, if $\varepsilon = 1$, the total profit of patent holders in the equilibrium is greater than that of a grand coalition except the case of $m = 12$. However, if $\varepsilon = 0.1, m = 12, 13, 14, 15, 16, 17, 18$, the total profit of a grand coalition is greater than the total profit in the equilibrium. Thus we can see that the sequential coalition formation cannot guarantees the total profit maximization, if $m > \frac{n+1}{2}$.

5 The patent thicket problem

5.1 The complementary patents and the patent thicket problem

In high technology industries, most technical developments are based on the earlier innovations. Thus, a new innovator might face a group of patents requiring him to pay royalties related to a certain technology. However, if some of the previous patents are complementary and stem from one common technology, a succeeding innovator must pay overlapping royalties for using the-earlier innovations. This situation represents the patent thicket problem. In this context, giving exclusive rights to every patent may be harmful to the objective of technical progress. As Cournot determined in 1838 when he studied the complementary problem, the patent thicket problem is more serious among complementary patents than among substitutive patents. For that matter, Shapiro (2000) suggests coordination among patent holders, such as a cross-license or a patent pool, in order to mitigate the patent thicket problem. The key idea is licensing complementary patents as a bundle, which might reduce the unnecessarily overcharged royalties to a more reasonable level.

5.2 The lower bound for bundle royalties

In this paper, we examine how much the bundle royalties can be reduced when forming a patent pool. To begin with, we assume that patent holders set their royalty as ε_i, the amount of cost reduction of the patent.[5] Thus, there is no incentive to use the other firm's patent. However, if a newcomer enters the Cournot market with his own patents which are based on the previous complementary patents, then he faces the patent thicket problem. More precisely, if the previous complemtentary patents are based on a common patent, say A, the newcomer must pay royalties for A as many times as the number of patents based on A.

To shed light on this problem, consider the following example. Suppose there exist m patent holders and $(n-m-1)$ firms without patents, and all m patents are

[5] If the cost reduction is relatively small and the innovator will act as a monopolist with no threat of entry, there is no need for licensing. Therefore $r_i = \varepsilon_i$ for $i = 1, \cdots, m$. More literature on the best way to license cost-reducing innovations, see Kamien (1992).

symmetric with regard to the amount of cost reduction, ε. However, an innovator enters the market with his own patent which can reduce the cost by η. This patent is based on the m previous patents which are complementary. For simplicity sake, suppose $m \leq \frac{n+1}{2}$ to make the equilibrium coalitiown structure be a grand coalition, which requires only one bundle royalties to be specified. Let us denote the bundle royalty to be R and the individual royalty to be ε, and compare the payoff for the patent holder between two situations: (1) the patent holders do not form a patent pool and each of them requires the newcomer to pay ε as royalties; and (2) the patent holders form a patent pool and charge the newcomer bundle royalties R.

First, consider the case where all patent holders remain single. The patent holders do not want to use the newcomer's patent because, under the perfect information, the newcomer would require almost η, as royalties. Thus, the payoff of each patent holder, say Φ_1, is the sum of his own profit and the royalties paid by the necomer.

$$\Phi_1 = \left(\frac{a-c+n\varepsilon-\eta-(m-1)\varepsilon}{n+1}\right)^2 + \left(\frac{a-c+n\eta-m\varepsilon}{n+1}\right) \times \varepsilon$$

Second, consider the case where all patent holders form one patent pool. For the same reason as the case above, there is no incentive for the patent pool members to use the newcomer's technology. However, if bundle royalties R is less than $m\varepsilon$, then using these patents is beneficial to all of the firms which do not have their own patents. The newcomer should pay for bundle royalty R, since his technology inherently relies on these previous technologies. Thus, the individual profit of pool members, say Φ_2, is the sum of their own profit and the royalties from the newcomer as well as external $(n-m-1)$ firms.

$$\Phi_2 = \left(\frac{a-c+(n-m+1)m\varepsilon-\eta-(n-m-1)m\varepsilon}{n+1}\right)^2 + \frac{R}{m} \times$$

$$\left\{\left(\frac{a-c+n\eta-m^2\varepsilon-(n-m-1)m\varepsilon}{n+1}\right)+\left(\frac{a-c+n\varepsilon-\eta-m^2\varepsilon-(m-n-2)m\varepsilon}{n+1}\right)\cdot(n-m-1)\right\}$$

From the above equations, we can find a sufficient condition to make bundle royalties lower than $m\varepsilon$. Let the lower bound for R be a minimum level of bundle royalties which satisfies $\Phi_1 \leq \Phi_2$.

Proposition 4 Let η be the cost reduction of newcomer's patent. If $\frac{n+1}{3} \leq m \leq \frac{n+1}{2}$ and $\eta \leq a-c+n\varepsilon+m\varepsilon(n+1-2m)$, then the lower bound for bundle royalties R is smaller than $m\varepsilon$.

Proof Since $m \leq \frac{n+1}{2}$, by the proposition 2, the grand coalition among patent holders is formed in the equilbirium. Let's denote Φ_1 be the individual payoff of patent holders when they don't form any patent pool, and Φ_2 to be the individual payoff when patent holder form a grand coalition.

$$\Phi_1 = \left(\frac{a-c+n\varepsilon-\eta-(m-1)\varepsilon}{n+1}\right)^2+\left(\frac{a-c+n\eta-m\varepsilon}{n+1}\right)\times\varepsilon,$$

$$\Phi_2 = \left(\frac{a-c+(n-m+1)m\varepsilon-\eta-(n-m-1)m\varepsilon}{n+1}\right)^2$$
$$+\frac{R}{m}\left\{\left(\frac{a-c+n\eta-m^2\varepsilon-(n-m-1)m\varepsilon}{n+1}\right)+\left(\frac{a-c+n\varepsilon-\eta-m^2\varepsilon-(m-n-2)m\varepsilon}{n+1}\right)\cdot(n-m-1)\right\}$$

However, if $\frac{n+1}{3} \leq m \leq \frac{n+1}{2}$ and $\eta \leq a-c+n\varepsilon+m\varepsilon(n+1-2m)$, then the following inequalities hold.

$$\left(\frac{a-c+n\varepsilon-\eta-(m-1)\varepsilon}{n+1}\right)^2 \leq \left(\frac{a-c+(n-m+1)m\varepsilon-\eta-(n-m-1)m\varepsilon}{n+1}\right)^2$$
$$\frac{a-c+n\eta-m\varepsilon}{n+1} \leq \frac{a-c+n\eta-m^2\varepsilon-(n-m-1)m\varepsilon}{n+1}+\left(\frac{a-c+n\varepsilon-\eta-m^2\varepsilon-(m-n-2)m\varepsilon}{n+1}\right)\cdot(n-m-1)$$

Defining the lower bound for R is a minimum level of bundle royalties which satisfis $\Phi_1 \leq \Phi_2$, the lower bound for $\frac{R}{m}$ is smaller than ε. Q.E.D.

The above proposition shows the main result of our model. It says that if the ratio between patent holders and the whole competitors lies within the certain interval, and the amount of cost reduction from the new patent is not too large, then charging royalties as a bundle is more profitable to patent holders. Thus, as desirable, forming a patent pool could mitigate the patent thicket problem, giving support to the Shapiro (2001) 's argument. Note that, for simplicity, we restrict our focus to the situation of $m < \frac{n+1}{2}$, which results in the grand coalition at the equilibrium.

Note that the lower bound for bundle royalties is the minimum value of bundle royalties which guarantees at least the same profits from noncooparation among patent holders. In other words, to ensure cooperation of all patent holders in forming a patent pool, the bundle royalties should satisfy this constraint.

To show a numerical example, assume that $n = 20$, $m = 9$, and $\varepsilon = 1$. The following table shows the lower bound for bundle royalties with respect to η. Assuming that the newcomer can use all of the previous technologies if she pay royalties, the new patent is valid only when it can effectively reduce the marginal price. For example, if the cost reduction from the new patent is less than what could be accomplished by using all previous patents, it is no use of R&D. In this context, we assume η to be equal to or greater than 9. The elements of the table, $R/m\varepsilon$, means the ratio between the lower bound for bundle royalties to the total payment for using all previous patents without patent pool. We can see that $R/m\varepsilon < 1$, that is, the bundle royalties can reduce the level of overlapping charged royalties by forming a patent pool, not harming the patent holders as well.

a-c	30	40	50
η	$R/m\varepsilon$	$R/m\varepsilon$	$R/m\varepsilon$
9	0.38	0.32	0.27
10	0.42	0.35	0.30
11	0.45	0.37	0.32
12	0.48	0.40	0.35
13	0.51	0.43	0.37
14	0.54	0.45	0.39
15	0.57	0.48	0.41
16	0.60	0.50	0.44
17	0.62	0.53	0.46
18	0.65	0.55	0.48
19	0.67	0.57	0.50
20	0.70	0.59	0.52

<Table 2>

The lower bound for bundle royalties

This shows that as the relative ratio of η to $a - c$ decreases, the minimum amount of bundle royalties goes down. And the bundle royalties goes down as the cost ruduction of the new innovation is relatively small. Even though η is 20 times larger than ε, the lower bound for the bundle royalties is less than $m\varepsilon$. Thus, forming a patent pool leads to lower the level of payments for using certain technologies, thus mitigate the patent thicket problem.

6 Conclusions

In this paper, we examine the possibility of cooperation among patent holders in the Cournot market. The game consists of two stages. In the first stage, firms form a patent pool sequentially through the alternating-offers bargaining game under the perfect information and in the second stage, they compete in the market. To rule out the anti-competitive effect, we assume that all patents complement each other, and to rule out the effect of allocation rule, we focus on the symmetric case in which all patents can reduce the marginal cost at the same amount. From these assumptions, we derive the worth of coalitions with the partition function form game and find the Markov-perfect equilibrium coalition structures.

In the symmetric case we can focus on the choice-of-sizes game to find the equilibrium. The coalition structure in the Markov-perfect equilibrium in this game is either one grand coalition or two asymmetric coalitions. The grand coalition among patent holders is formed if the number of patent holders is less than almost half of the number of total competitors in the Cournot market. Otherwise, they form two asymmetric patent pools where the dominant patent pool contains $\left[\frac{n+1+2m}{4}\right]$ or $\left[\frac{n+1+2m}{4}\right]+1$ members. If $m < \frac{n+1}{2}$, then the equilibrium coalition structures maximize the total profit of patent holders, which does not hold if $m > \frac{n+1}{2}$. Finally, we examine whether or not forming a patent pool mitigates the patent thicket problem, and find the lower bound for bundle royalties.

The assumption that all patents are perfectly complementary seems strong. However, in the context of the patent thicket problem, coordination among patent holders is desirable as long as they complement. In future studies, we can extend investigation into substitute patents as well as partially complementary patents.

Finally, since many patents enhance a good's competitive power by differentiating its quality, extending our study to heterogenous goods would be an important topic.

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