An Economic Theory of Judicial Torture

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Abstract

Judicial torture to elicit information or confession is a very common practice in pre-modern societies, either in the East or the West. Moreover, sometimes it was applied not only to suspects, but also to witnesses and plaintiffs as well. This paper proposes a theory of judicial torture. It is shown that if the judge aims to balance type I and type II errors in decision-making, and relatively little information is revealed during investigation, then torture can maximize social welfare in that it forces the suspects (both innocent and guilty) to confess for fear of being tortured. In that case torturing the witnesses might also be welfare-improving, as it helps to screen the cases so that only those with greater merits come into the court.

Keywords: Torture; Type I and Type II errors; Evidence.

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1 Introduction

Judicial torture for the purpose of eliciting information was a common practice in pre-modern societies. In the West, it emerged in Greek law and continued in Roman law. Its history can then be traced through the Middle Ages, down to the legal reforms of the eighteenth century and the abolition of torture in criminal legal procedure in the early nineteenth century in most parts of Europe. (Peters 1985, p.5.) In China, judicial torture formed an important part of the imperial legal codes from the first Empire (Qin dynasty, 221-207 BC) to the last Qing dynasty (1644-1911). (Shen 1985.) Various forms of judicial torture were also widely implemented in Moslem, African and various Asian societies before our time. (Lea 1971, 203-208.) It is indeed hard to find any pre-modern society that did not rely on torture for the gathering of evidence in judicial proceedings.

The modern man, for whom judicial torture is not only immoral but also irrational, may imagine that it was inflicted exclusively on the accused to obtain his confession. Nothing is further from the truth. Judicial torture in pre-modern times was applied not only to the suspect, but also to witnesses, and even to the plaintiff. Witnesses were tortured already in Greek and Roman cases, and continued to be so in the medieval period and until the late 18th century. (Peters 1985, p.18, p.69.) Even the accuser or plaintiff would undergo an ordeal to substantiate his veracity or be tortured when he was unable to make good his accusation. In Roman law, the accuser could be exposed to the lex talionis in case he failed to prove the justice of the charge. (Lea 1971, p.333.) In primitive Russian laws, the accuser was obliged to undergo the ordeal of red-hot iron if he could not substantiate his case with witnesses. Archbishop Hincmar of Rheims of the 9th century required that cases of complaint against priests be supported by seven witnesses, of whom one must be tortured to prove the truth of his companions’ oath, as a wholesome check upon
perjury and subornation. (Lea 1971, p.290.)

The same principles could be observed in China. The legal code of the Qing dynasty clearly allowed torture to be used not only on the accused, but also on “secondary suspects and to witnesses as well as to principals”. (Bodde and Morris 1967, p.97.) In fact, the tradition could be traced back to the Tang code of 653 AD that constituted the basis for all subsequent imperial law codes. According to Tang law, if the accused insisted on his innocence even after having suffered the maximum amount of torture allowed by the law, the plaintiff would in turn be tortured. (Shen 1985, p.511.) Moreover, since at least the Tang dynasty, a plaintiff who bypassed the immediate authority and presented his complaints to higher administrative levels was required to undergo torture before the examination of the case (Xue 1998, pp.636-639), again as a check upon perjury and subornation, or as a measure to discourage such actions. During the early years of the Ming dynasty (1368-1644), frequent complaints brought directly to the capital prompted the government to implement an extremely severe punishment - banishment to the frontier - on plaintiffs, even at times on those who could substantiate their cases, to control the number such acts. (Shen 1971, p.1142.) In brief, judicial torture in pre-modern societies was legitimately applied almost universally to suspects, witnesses, and plaintiffs for various reasons before the mid-nineteenth century.

Starting from around mid-18th century, the practice of judicial torture in the West was gradually replaced by a system which was based on evidence. Its abolition was a long and gradual process that lasted from the mid-18th to the early 19th centuries. The conventional historical explanation of this abolition movement relies heavily on the influence of the humanists of the Enlightenment. However, experts on jurisprudence usually find this explanation too loose and prefer to explain the phenomenon by changes in the judicial system itself. According to Langbein, the
abolition was largely due to the emergence of the new law of proof. During this period, the courts gradually imposed the new and less rigorous punishments according to a less strict standard of proof, one of persuasion rather than certainty. Since certainty was no longer the only requirement to put an accused in prison, a lesser punishment, torture became unnecessary. “Only when confession evidence was no longer necessary to convict the guilty could European law escape its centuries of dependence on judicial torture.” (Langbein 1983, pp.1555-1556.)

An indirect but equally important factor in the process of the abolition of judicial torture in the West, especially towards its end, should be the application of scientific knowledge in criminal investigation beginning in the early 19th century. Experts in criminalistics agree that as early as in the 1820s, the basis was laid for the introduction of empirical science into criminal justice and for the redefinition of criminalistics. During the 19th century, one witnessed important breakthroughs in the application of scientific knowledge in criminal investigations including the use of precise measurements of human body structure (anthropometry) and fingerprinting. At the same time, 19th-century science became a vocation involving a growing scientific community drawn from the middle and lower classes. As serving as courtroom experts became an attractive source of income for professional scientists, chemists were commonly employed as such by the mid-century. The application of scientific methods in criminal investigations had obviously reached a mature stage in the late 19th century when Hans Gross published the first classic in the specialty: Criminal Investigation in 1883. With such technological breakthroughs, past human activity can be specified without relying exclusively on the confessions of the guilty or witnesses, often obtained by torture in earlier times. (Parker 1983, 432-433; Fullmer 1980, p.27).

In this paper, we try to make sense of the existence and abolition of judicial
torture by using a unified model. Our explanation is based on the modern economic theory of information. It is assumed that the objective of the judge is to balance the trade-off between the type-I (erroneously convicting an innocent suspect) and the type-II (erroneously releasing a guilty suspect) error. If during investigation very little information is revealed (because of, say, technological retardation in fact-finding), then sometimes torturing the suspect to the extent that all confess (regardless of guilty or not) maximizes social welfare. We also show that as the information revealed during investigation increases, the advantage of torture will decrease. In that case a system based on evidence, rather than confession, will result in higher social welfare than a system based on torture. This explains why in the late 18th, a modern judicial system which enables the judge to convict a suspect solely on evidence without confession gradually overtook judicial torture.

The paper also shows that, if the precision of information revealed during investigation is low, there will also be advantage in torturing the witness or plaintiff. The reason is that it helps to screen the cases that come into the court. By torturing the plaintiff or witness, the judge can make sure that only cases with higher merits will come into court, in which he will sometimes torture to illicit confession. This helps to reduce type-I error, and thus increases social welfare.

2 The Model

A crime has been committed and a suspect is brought to the court. He is either guilty ($\theta = G$) or innocent ($\theta = I$), which is his private information. The judge needs to make a verdict on whether he is guilty or not. The prior belief (which may be based on the information provided by a witness or the plaintiff) that the suspect is guilty is $q$, $0 < q < 1$. That is, $\text{Prob}(\theta = G) = q$. 
In order to convict a suspect, confession from the suspect is required. For that purpose the judge is allowed to torture the suspect. We model this as the follows. The suspect is given an option whether to confess. If he does, a legal penalty of the crime causing disutility $P$ is applied. If not, the judge will investigate the case by collecting an evidence $x$ regarding whether he is guilty. Based on the value of $x$ collected, the judge decides whether to release the suspect. If he does, then the suspect has a utility 0. If he decides not to, then he tortures the suspect to elicit confession. Let $T$ denote the disutility for the suspect of being tortured. If the suspect does not confess after torture, he is released with a disutility $T$. If he confesses, his total disutility is $T + P$.

The judge’s investigation can only improve the precision of his information, but not resolve the uncertainty on whether the suspect is guilty. The precision of investigation is modelled in the following way: If the suspect is indeed guilty, then the density function of $x$ is $f_G(x)$. If he is innocent, then the density function for $x$ is $f_I(x)$. Signal $x$ is informative in the sense that, the greater its value, the more likely that the suspect is guilty. The standard assumption to capture this is that $f_I(x)$ and $f_G(x)$ satisfy the monotone likelihood ratio property (MLRP); that is, $f_I(x)/f_G(x)$ is a decreasing function of $x$. Let $F_G(x)$ and $F_I(x)$ be the distribution functions for $f_G(x)$ and $f_I(x)$, respectively. MLRP property implies that $F_G(x) < F_I(x)$ for all $x$. In other words, $F_G(x)$ first-order stochastically dominates $F_I(x)$. For technical reason that will become clear shortly, we further assume that:

$$
\text{A1. } \lim_{x \to -\infty} \frac{f_I(x)}{f_G(x)} = \infty, \text{ and } \lim_{x \to -\infty} \frac{f_I(x)}{f_G(x)} = 0.
$$

A1 implies that when the judge collects a very high (low) value of $x$, that means it is infinitely more likely that the suspect is guilty (innocent) than otherwise.

Since investigation is imperfect, there are two possible errors that the judge can make in the procedure. First, he might wrongfully convict a suspect who is actually
innocent. We call this a type I error. Second, he might wrongfully release a suspect who is actually guilty. We call this a type II error. Assume that type I error causes a loss of social welfare by an amount $L_1$, and type II error by an amount $L_2$. Also assume that if a level of torture $T$ is imposed on the suspect, it causes a social loss of $kT$, $k < 1$.

The events proceed in the following order: (1) Natures determines whether the suspect is guilty or not (i.e., whether $\theta = G$ or $\theta = I$). (2) The suspect decides whether to confess. If he confess, a legal penalty of $P$ is applied. If he does not, (3) the judge conducts the investigation by collecting an evidence $x$. Based on the evidence, he decides whether to torture the suspect. If not, the suspect is released. (4) If tortured, the suspect decides whether to confess (and penalized by $P$, in addition to the disutility of torture, $T$), or not (and is released).

## 3 The Game of Torture

We assume that in deciding whether to torture a suspect, the judge aims to maximize social welfare. In our context, this amounts to minimizing the loss of social welfare in the procedure. Suppose that for cases that enter the court, the prior of the judge that the suspect is guilty is $q$. Then the probability of making a type I error is $1 - q$ if the suspect is convicted, and the probability of making a type II error is $q$ if the suspect is released. The expected social loss when a suspect is tortured to confess is thus $(1 - q)L_1 + kT$; and the expected social loss when a suspect is released is $qL_2$. Note that if the objective of the judge is to minimize social loss, then once he tortures a suspect he must torture him to the extent to obtain confession. This is

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1In later part of the paper, we will investigate how the plaintiff or witness helps to screen the cases by deciding whether to bring the cases to court. As a result, the value of $q$ can be different from $\theta$. 

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because if he tortures a suspect and then releases him without a confession, then he could have released him without any torture and increased the social welfare. This implies that the judge either tortures to the extent that the suspect confesses, or does not torture at all. There is therefore no need to consider the case in which the judge tortures and the suspect does not confess.

### 3.1 The Optimal Decision of the Judge

Since the suspect’s type is his private information, his decision will be based on whether he is innocent or guilty, and this decision will play a role in determining the posterior of the judge. Suppose in equilibrium, the innocent suspect denies the crime with probability \( \nu_I \), and the guilty denies the crime with probability \( \nu_G \). When the suspect does not confess, the judge can conduct an investigation to collect an evidence \( x \). Given the evidence and the strategy of the suspect, the posterior that the suspect is guilty is:

\[
\hat{q}(\theta = G | x, \nu_G, \nu_I) = \frac{q\nu_G f_G(x)}{q\nu_G f_G(x) + (1-q)\nu_I f_I(x)}.
\]

Based on this posterior, if the judge decides to torture the suspect, the expected social loss is \((1-\hat{q}(x))L_1+kT\). If she decides not to torture the suspect, the expected loss is \(\hat{q}(x)L_2\). Therefore, the judge tortures the suspect if and only if

\[
\frac{(1-q)\nu_I f_I(x)}{q\nu_G f_G(x) + (1-q)\nu_I f_I(x)} L_1 + kT \leq \frac{q\nu_G f_G(x)}{q\nu_G f_G(x) + (1-q)\nu_I f_I(x)} L_2. \tag{1}
\]

Note that by A1, the left-hand (resp. right-hand) side of (1) approach \(L_1+kT\) (resp. 0) as \(x \to -\infty\), and approach \(kT\) (resp. \(L_2\)) as \(x \to \infty\). Note that if \(L_2 \leq kT\), then the potential loss of social welfare in torturing the suspect is so large compared with

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2We have thus implicitly assumed that the posterior of the judge remains constant during the torture. That is, no matter how much a suspect is tortured, as long as he persists in claiming innocent, then the posterior that the suspect is guilty remains at \(\hat{q}\).
the potential gain in avoiding type II error, so that the judge will never torture. In
that case torture will never be used, and its function in balancing type I and type II
errors is non-existent. We will thus concentrate on the case when torture is of use
by making the following assumption:

**A2.** \( L_2 > kT \).

We have the following lemma.

**Lemma 1.** Given \( x, \nu_G, \nu_I \), there exists a unique threshold, \( x_c(\nu_G, \nu_I) \), such that
the judge tortures the suspect if and only if \( x \geq x_c(\nu_G, \nu_I) \).

**Proof.** From MLRP we know that the left-hand side of (1) is increasing in \( x \), and
the right-hand side is decreasing in \( x \). This, together with assumption **A1**, implies
that there exists \( x_c \in (-\infty, \infty) \) such that the right-hand (resp. left-hand) side of
(1) is smaller (resp. greater) than the left-hand (resp. right-hand) side for \( x < x_c \)
(resp. \( x > x_c \)). \( \square \)

According the Lemma 1, the decision of the judge after he collects the evidence
is in a very simple form: Torture the suspect if and only if the collected evidence \( x \)
is greater than a certain threshold \( x_c \). The judge therefore chooses an optimal value
of threshold, \( x^*_c \), that minimizes expected social loss:

\[
\min_{x_c} \int_{-\infty}^{x_c} \left[ \frac{q \nu_G f_G(x)}{q \nu_G f_G(x) + (1 - q) \nu_I f_I(x)} L_2 \right] dy + \int_{x_c}^{\infty} \left[ \frac{(1 - q) \nu_I f_I(x)}{q \nu_G f_G(x) + (1 - q) \nu_I f_I(x)} L_1 + kT \right] dy;
\]

or equivalently,

\[
\min_{x_c} W^c(x_c; q, \nu_G, \nu_I) = q \nu_G F_G(x_c)(L_2 - kT) + (1 - q) \nu_I (1 - F_I(x_c))(L_1 + kT). \quad (2)
\]

Given \( \nu_G \) and \( \nu_I \), the optimal value of \( x_c \) must satisfy the following first-order condition:

\[
\frac{f_I(x^*_c)}{f_G(x^*_c)} = \frac{q \nu_G (L_2 - kT)}{(1 - q) \nu_I (L_1 + kT)}. \quad (3)
\]
According to MLRP, we have $\frac{\partial x^*_c}{\partial \nu_G} \leq 0$ and $\frac{\partial x^*_c}{\partial \nu_I} \geq 0$. Thus, it is more likely for the judge to torture the suspect when the guilty (the innocent) confesses with lower (higher) probability.

### 3.2 The Optimal Decision of the Suspect

For the guilty suspect, if he confesses he will receive a penalty $P$. If he does not, then he will be tortured with probability $1 - F_G(x^*_c)$ and suffers $T + P$. Thus, the guilty suspect’s problem is

$$
\min_{\nu_G} \ (1 - \nu_G)P + \nu_G \left[ 1 - F_G(x^*_c(\nu_G, \nu_I)) \right] (T + P). \tag{4}
$$

Similarly, the innocent suspect’s problem is:

$$
\min_{\nu_I} \ (1 - \nu_I)P + \nu_I \left[ 1 - F_I(x^*_c(\nu_G, \nu_I)) \right] (T + P). \tag{5}
$$

In the extreme case when the investigation is perfectly informative, the judge can torture the suspect merely based on the evidence found without causing any social loss. In that case the guilty suspect will always confess, since otherwise he will be forced to confess by torture. On the other hand, the innocent suspect never confesses, since he knows that he will be acquitted after investigation. This means that a fully separating equilibrium can be supported. The more interesting case, however, occurs when the investigation is not perfectly informative.

In the following proposition, we show that there exist three types of equilibria. In the first two cases, the guilty suspect confesses with a positive probability, and the innocent always denies the crime. In the third case, both types always confess.
Proposition 1. As shown in Figure 1, the equilibrium in this game is:

1. \( \nu^*_I = \nu^*_G = 1 \) in Region I.

2. \( \nu^*_I = 1 \) and \( 0 < \nu^*_G < 1 \) in Region II.

3. \( \nu^*_I = \nu^*_G = 0 \) in Region III, which is supported by an off-equilibrium belief \( \tilde{q} > 0 \).

The division line \( M_1 \) between Regions I and II satisfies \( \frac{P}{T + P} = \lambda(x^*_c(1)) \), where \( \lambda \equiv 1 - F_G - \frac{f_I}{(1/F_G)^r} \); and the line \( M_2 \) between Regions II and III satisfies \( \frac{P}{T + P} = [1 - F_I(x^*_c(\tilde{q}))] \).

Proof.

1. For the first case, the first-order condition for \( \nu^*_G = 1 \) is:

\[
-\frac{P}{T + P} + [1 - F_G(x^*_c)] - \nu^*_G f_G(x^*_c) \frac{\partial x^*_c}{\partial \nu_G} < 0. \tag{6}
\]

However, since \( F_I(\cdot) > F_G(\cdot) \) by FOSD, and \( \frac{\partial x^*_c}{\partial \nu_G} \leq 0 \), \( \frac{\partial x^*_c}{\partial \nu_I} \geq 0 \) from (3), (6) implies

\[
-\frac{P}{T + P} + [1 - F_I(x^*_c)] - \nu^*_I f_I(x^*_c) \frac{\partial x^*_c}{\partial \nu_I} < 0. \tag{7}
\]

Therefore, \( \nu^*_I = 1 \).

Since the sum of the last two terms in (6) and (7) is the total change in the probability of torture as \( \nu_G \) (\( \nu_I \)) changes, which is always less than 1, this equilibrium must exist when \( \frac{P}{T + P} \) is sufficiently large, and \( \frac{q(L_2 - kT)}{(1-q)(L_1 + kT)} \) is sufficiently small. Otherwise, for example, if \( P \rightarrow 0 \), the guilty suspect will always confess, i.e. \( \nu_G = 0 \); and if \( L_2 \rightarrow \infty \) or \( (L_1 + kT) \rightarrow 0 \), the standard \( x^*_c \rightarrow -\infty \), and the judge will always torture the suspect, so that \( [1 - F_G(x^*_c)] \rightarrow 1 \). In that case, the guilty suspect will always confess as well.

As shown in Figure 1, suppose \( \nu^*_G = 1 \) at point A in Region I. We will show that \( \nu^*_G = 1 \) as well at another point located lower-right to A. First of all, given the same
\[
\frac{L_2-kT}{L_1+kT}
\]

Figure 1. Equilibrium with Torture

\(\frac{L_2-kT}{L_1+kT}\), it is obvious that (6) is still true at another point with a larger \(\frac{P}{T+P}\) than at point \(A\). Second, consider another point with a smaller \(\frac{L_2-kT}{L_1+kT}\) but the same \(\frac{P}{T+P}\). Differentiating (3) with respect to \(\nu_G\), we have

\[
\frac{\partial x^*}{\partial \nu_G} = \frac{q(L_2-kT)}{(1-q)\nu_G(L_1+kT)'} \left( \frac{f_I}{f_G} \right)' < 0
\]

by MLRP, and thus the first-order derivative can be rewritten as a function of \(x^*_c(\nu_G)\):

\[
-\frac{P}{T+P} + \lambda(x^*_c(\nu_G))
\]

where \(\lambda \equiv 1 - F_G - \frac{f_I}{(f_I/f_G)'}) > 0\). The second-order condition implies that \(\lambda(\cdot)\) is decreasing in \(x^*_c\) (and increasing in \(\nu_G\)). Thus, when \(\frac{L_2-kT}{L_1+kT}\) is smaller, and the guilty suspect again chooses \(\nu_G = 1\), the induced \(x^*_c\) becomes larger, so that the derivative will still be negative. Therefore, the solution is at corner again.

2. In Region II, \(\nu^*_G\) is interior and \(\nu^*_I = 1\). Parameters in this region satisfy

\[
-\frac{P}{T+P} + \lambda(x^*_c(\nu_G)) = 0
\]

(8)
Given point B in this region, another point located lower-right to B will correspond to a higher $\nu_G^*$. The reason is as follows. When $\frac{L_2 - kT}{L_1 + kT}$ is fixed and $\frac{P}{T + P}$ is larger, in order to maintain (8), $\lambda(x_c^*(\nu_G))$ has to be increased, which can be achieved by choosing a larger $\nu_G^*$. On the other hand, if $\frac{P}{T + P}$ is fixed and $\frac{L_2 - kT}{L_1 + kT}$ is smaller, under the original $\nu_G$, a larger $x_c^*$ will be selected, such that $\lambda(x_c^*(\nu_G))$ is lower. In order to make (8) hold, a larger $\nu_G^*$ has to be selected.

The line $M_1$ divides Regions I and II, where it is optimal to choose $\nu_G^* = 1$. That is, parameters along this line satisfy $\frac{P}{T + P} = \lambda(x_c^*(1))$. $M_1$ obviously has a positive slope.

3. For the third case, we first show the following lemma:

**Lemma 2.** That $\nu_G = 0$ and $\nu_I > 0$ cannot be a profile played on the equilibrium path.

*Proof.* Suppose that $\nu_G^* = 0$ and $\nu_I^* > 0$. Then the posterior while seeing the suspect denying he crime will be $\hat{q} = 0$, that is, he is always innocent. Then by (3), the judge’s best response is never to torture the suspect, i.e. $x_c^* \to \infty$. However, the first-order condition for the guilty suspect to choose $\nu_G = 0$

$$-rac{P}{T + P} + [1 - F_G(x_c^*)] - \nu_G f_G(x_c^*) \frac{\partial x_c^*}{\partial \nu_G} > 0$$

(9)
cannot hold since the last two terms approach to 0 in this case, so that it is never optimal for the guilty suspect to select $\nu_G^* = 0$ in the first place. $\square$

According to this lemma, the only possible equilibrium where $\nu_G^* = 0$ is such that $\nu_I^* = 0$ as well. That is, both types always confess. In this case, torture is off the equilibrium path. The standard $x_c^*$ will be determined by some off-equilibrium belief $\hat{q}$. However, it must be $\hat{q} > 0$, since otherwise, $x_c^* \to \infty$, and it is never optimal for the guilty (innocent) suspect to choose $\nu_G = 0$ ($\nu_I = 0$).
We can see that the innocent suspect will choose either \( \nu^*_I = 1 \) or \( \nu^*_I = 0 \). The condition for \( \nu^*_I = 0 \) is
\[
- \frac{P}{T + P} + [1 - F_I(x^*_c(\tilde{q}))] > 0. \tag{10}
\]
where \( x^*_c \) is determined at \( \frac{f_I(x^*_c)}{f_G(x^*_c)} = \frac{\tilde{q}(L_2 - kT)}{(1 - \tilde{q})(L_1 + kT)} \). Such an equilibrium exists when \( \frac{P}{T + P} \) is smaller and \( \frac{\tilde{q}(L_2 - kT)}{(1 - \tilde{q})(L_1 + kT)} \) is larger, so that the cost to confess is low and/or the probability of torture is high enough for both types to always confess.

Suppose that \( \nu^*_I = 0 \) at point \( C \) in Region III. We will show that \( \nu^*_I = 0 \) as well at another point located upper-left to point \( C \). It is easy to see that given \( \frac{\tilde{q}(L_2 - kT)}{(1 - \tilde{q})(L_1 + kT)} \), (10) is still true at another point with a smaller \( \frac{P}{T + P} \) than at point \( C \).

Likewise, given \( \frac{P}{T + P} \), (10) is again true at another point with a larger \( \frac{\tilde{q}(L_2 - kT)}{(1 - \tilde{q})(L_1 + kT)} \), because now \( x^*_c \) is lower and thus \( 1 - F_I(x^*_c) \) is larger.

The line \( M_2 \) divides Regions II and III, where the innocent suspect is indifferent between \( \nu^*_I = 0 \) and \( \nu^*_I = 1 \), and the guilty suspect is also indifferent between \( \nu_G = 0 \) and an interior solution \( \nu^*_G \). That is, parameters along \( M_2 \) satisfy \( \frac{P}{T + P} = [1 - F_I(x^*_c(\tilde{q}))] \). \( M_2 \) obviously has a positive slope.

There are three effects when the guilty suspect increases the probability to deny the crime. First, the probability of paying the penalty is reduced. Second, the probability of torture increases because he denies the crime more likely; and third, since the standard the judge sets for torture decreases, the probability of practicing torture increases furthermore. If the first effect dominates (e.g. when penalty is high enough and/or the cost of torture is very small), the guilty suspect denies the crime with some positive probability. Otherwise, he always confesses.

The intuition why the innocent suspect always denies the crime in the first kind of equilibrium is because the judge will share the blame between the guilty and the innocent when the guilty one denies the crime with some probability, so that the
marginal benefit of increasing the probability to deny the crime is larger for the innocent suspect than the guilty one.

As can be imagined, eliciting confession through torture is a costly way of seeking justice because once torture is applied, not only the guilty but also the innocent suspects are forced to confess to crimes. It is thus more likely to be used either when social loss of type I error is small (or social loss of type II error is large), or the investigation is very uninformative. The modern time, with the help of more scientific device (such as blood type test and finger print take), can enormously increase the accuracy of the investigation. As a result, the practice of torture is lost to the evidence-based system. Under this system, the court will first set a standard of proof. If the evidence collected surpasses that standard, then the judge is allowed to convict the suspects without soliciting their confession. In the following section, we model this system as a sampling process, and find the optimal standard of proof that maximizes the social welfare.

3.3 The Evidence-based System

Under the evidence-based system, the judge convicts the suspect purely based on the evidence she gathers. His goal is to choose an optimal standard of proof, $x_e$, to minimize the expected social loss, in that he convicts the suspect if $x \geq x_e$, and acquits him if $x < x_e$. Thus, the judge’s minimization problem is:

$$\min_{x_e} W^e(x_e; q) = (1 - q)[1 - F_G(x_e)]L_1 + qF_G(x_e)L_2.$$  \hspace{1cm} (11)

The following proposition shows the condition for an optimal standard of proof under the evidence-based system.

**Proposition 2.** The unique optimal standard of proof $x_e^*$ is selected such that:

$$\frac{f_I(x_e^*)}{f_G(x_e^*)} = \frac{qL_2}{(1 - q)L_1}.$$  \hspace{1cm} (12)
Proof. It is straightforward from the first-order condition, as long as sampling is not perfectly informative. There must exist such a unique $x_c^*$ because of A1 and MLRP.

The judge will choose a threshold that balances the likelihood to the relative cost of type II error and type I error. According to (12) and MLRP, when the judge believes that the suspect is more likely to be innocent (a lower $q$), the cost of type II error is lower (a smaller $L_2$), or the cost of type I error is higher (a larger $L_1$), he will set a higher standard, which means he is more lenient.

Comparing the equilibrium standards under these two systems, we can see that the confession-based system uses a more lenient standard to convict a suspect as in the following result:

**Corollary 1.** $x_e^* \leq x_c^*$.

*Proof. It is straightforward by comparing (3) to (12), and also by MLRP.*

This result has some interesting implication. A large cost of torture itself may lead the judge to be more lenient under the confession-based system; however, even if there is no social loss of torture (i.e. $k = 0$), the judge still behaves more leniently, because if torture is on the equilibrium path, there is some probability that the guilty suspect confesses, so that a suspect who does not confess is more likely innocent.

### 3.4 Welfare Analysis

In this section, we compare the difference in welfare under these two systems. In the *ex ante* sense, the welfare loss under the evidence-based system is:

$$W^e = (1 - q)[1 - F_I(x_e^*)]L_1 + qF_G(x_e^*)L_2.$$
Under the confession-based system, in the equilibrium where $\nu^*_I = 1$ and $0 < \nu^*_G \leq 1$, the welfare loss is:

$$W^c = (1-q)[1-F_I(x^*_c)]L_1 + q\nu^*_G F_G(x^*_c)L_2 + \{ (1-q)[1-F_I(x^*_c)] + q\nu^*_G [1-F_G(x^*_c)] \} kT.$$ 

In the equilibrium where $\nu^*_G = \nu^*_I = 0$, torture is not on the equilibrium path, and the welfare loss under the confession-based system is $(1-q)L_1$.

Figure 2 demonstrates the comparison in welfare of these two systems. In what follows, we discuss two cases separately.

**Regions I and II: Suspects do not always confess.**

The difference in welfare loss in these two regions is:

$$\Delta W \equiv W^c - W^e = (1-q)[F_I(x^*_e) - F_I(x^*_c)]L_1 + q[\nu^*_G F_G(x^*_c) - F_G(x^*_e)]L_2$$

$$+ \{ (1-q)[1-F_I(x^*_c)] + q\nu^*_G [1-F_G(x^*_c)] \} kT. \quad (13)$$

First of all, there is a cost of torture under the confession-based system, as appeared in the last term. However, compared to the evidence-based system, the confession-based system can induce a lower type I error, since the first term in (13) is always non-positive, according to the fact $x^*_e \leq x^*_c$. Whether this system can reduce the type II error or not is uncertain: on the one hand, a higher standard $x^*_c$ leads to a higher type II error; however, since the guilty suspect may confess with some probability, the chance of committing it is reduced.

Other things being equal, suppose that now $P$ increases so that $\frac{P}{T+P}$ increases. We have

$$\frac{d\Delta W}{dP} = q\{ F_G(x^*_c)L_2 + [1-F_G(x^*_c)]kT \} \frac{d\nu^*_G}{dP} > 0.$$ 

This means that when the cost of penalty relative to cost of torture increases, the evidence-based system is more welfare-improving. Accordingly, in order to keep $\Delta W = 0$, a higher $\frac{P}{T+P}$ is accompanied with a higher $\frac{L_2-kT}{L_1+kT}$, as shown in Figure 2.
The confession-based system is better than the evidence-based system (i.e. $\Delta W < 0$) in the area above this division line, and worse under it (i.e. $\Delta W > 0$).

**Region III:** Suspects always confess.

The difference in welfare loss in this region is:

$$\Delta W = (1-q)F_I(x^*_c)L_1 - qF_G(x^*_c)L_2.$$  \hspace{1cm} (14)

Since $x^*_c$ does not depend on the level of $\frac{P}{P+T}$, $\Delta W$ is not affected as it changes. When:

$$\frac{qL_2}{(1-q)L_1} = \frac{F_I(x^*_c)}{F_G(x^*_c)},$$  \hspace{1cm} (15)

then $\Delta W = 0$. That is, there is a critical value of $\frac{L_2}{L_1}$ or $\frac{L_2-kT}{L_1+kT}$ such that the confession-based system is better in the area above the critical value, as shown in Figure 2 as well. We summarize the above analysis in the following proposition.

\hspace{1cm}

**Figure 2. Welfare Analysis**
Proposition 3. The confession-based system yields a higher social welfare than the evidence-based system does if the cost caused by type II error is sufficiently larger than the cost caused by type I error, or the cost of torture is sufficiently small.

The reason for this result is intuitive. When the cost caused by type II error is relatively larger than the cost caused by type I error, then since the guilty suspect confesses sometimes because of the fear of being tortured, the judge can in fact save more in the cost of type II error compared with the evidence-based system. On the other hand, if the cost by type I error is relatively larger than the cost by type II error, then since the innocent suspect confesses sometimes, the society suffers from a large cost by type I error. Besides, it is obvious that the system based on torture is more efficient if the cost of torture is small.

Therefore, although it causes some welfare loss, torture has an advantage in that it forces the guilty suspect to confess. As we will see in the next section, this advantage is even more substantial when information is less precise. This result provides a rationale for torture being historically prevailing in pre-modern societies.

The merit of torture is even more apparent from the following aspect in that it is always dominated if both suspects do not confess at all. In Figure 2, we can see that it is always the case $\Delta W > 0$ in Region $I$, as we argue in the next result.

Corollary 2. If both suspects never confess, the confession-based system is always worse than the evidence-based system.

Proof. Given that the suspects never confess, we start with the situation where $k = 0$. Then $\Delta W = 0$ since $x_c^* = x_e^*$ (from (3) and (12)), an thus, these two regimes are indifferent.

However, generally, $k > 0$. Taking the derivative of (13) with respect to $k$, we
have:
\[
\frac{d\Delta W}{dk} = \{ (1 - q)[1 - F_I(x^*_I)] + q\nu^*_G[1 - F_G(x^*_G)] \} T > 0.
\] (16)
supposing that \( \nu^*_I = \nu^*_G = 1 \) remains the case as \( k \) changes (for instance, when \( P \) is large). That is, when \( k \) is greater than 0, it always yields a lower welfare if the judge convicts the suspects through torture.

The basic tradeoff is that the judge not only wishes to minimize the expected social cost caused by torture, but also aims to provide the suspects incentives to signal themselves. When both suspects never confess, the judge cannot distinguish them. Conviction will be merely based on torture, and these two regimes converge when there is no cost to torture the suspect. However, if torture causes some social costs, and both suspects still do not want to confess, the confession-based system creates no benefit in signalling and thus is dominated.

4 Improving the Information

In this section, we discuss how the changing in information structure affects the relative performance of these two systems. Suppose that the technology in investigation improves, so that sampling becomes more precise. Will the conviction based on evidence become a better scheme in general?

Suppose that there is another information structure with distribution \( H_\theta, \theta \in \{ I, G \} \), which also satisfies MLRP. The problem of what it means to have better information is classical since Blackwell (1951, 53); however, there seems no systematic method for deciding it. Fortunately, it is known that for dichotomies (experiments in which there are only two possible distributions over states) that satisfy MLRP, several different classes of problems yield the same information ordering in the following sense: \( H \) is more informative than \( F \) if and only if \( H \) yields tests more
powerful than \( F \) for each size.\(^3\) Namely, given a size \( \alpha \), the power of the most powerful test is given by

\[
\beta_F(\alpha) = 1 - F_G(F_I^{-1}(\alpha))
\]

Then \( H \) is more informative than \( F \) if and only if for each size \( \alpha \), the power of the most powerful test is greater under \( H \) than \( F \), that is,

\[
\beta_H(\alpha) \geq \beta_F(\alpha), \text{ for } \alpha \in [0, 1]. \tag{17}
\]

This basically states that \( H \) is more spread out than \( F \).

Under the new information structure \( H \), we denote the optimal standard of proof based on evidence by \( x_e^{**} \). Given \((L_1, L_2, k, T)\), we have the following result:

**Lemma 3.** When the judge uses the evidence-based system, the welfare under \( H \) is higher than under \( F \) when \( H \) is more informative than \( F \).

**Proof.** Applying the condition in (17), it is easy to see that:

\[
(1 - q)[1 - F_I(x_e^*)]L_1 + qF_G(x_e^*)L_2 \geq (1 - q)[1 - H_I(x_e^*)]L_1 + qH_G(x_e^*)L_2
\]

\[
\geq (1 - q)[1 - H_I(x_e^{**})]L_1 + qH_G(x_e^{**})L_2. \tag{18}
\]

The first inequality is due to the fact that \( H \) is more informative, and the second inequality is due to optimality.

In what follows, we will show that if the information is better, the evident-based system is more welfare-improving than the confession-based system. In other words, as the information revealed increases, the advantage of torture decreases. Denote \( \Delta_FW \) (\( \Delta_HW \)) the difference in welfare loss between these two system under information \( F \) (\( H \)). We have the following important result:

\(^3\)See Lehmann (1988) and Jewitt (1997).
Proposition 4. Given \((L_1, L_2, k, T)\), suppose that \(\Delta_F W = 0\) and \(H\) is more informative than \(F\). Then \(\Delta_H W > 0\).

Proof. Denote the probability of the suspects denying the crime under information \(H\) by \(\nu^{**}_I\) and \(\nu^{**}_G\) respectively. In what follows, we separate two different cases again:

Lemma 4. If \(\nu^{**}_I = \nu^{**}_G = 0\), then \(\Delta_F W = 0\) implies that \(\Delta_H W > 0\).

Proof. In this case, \(W^c = (1 - q)L_1\), so under \(H\):

\[
\Delta_H W = (1 - q)H_I(y^*_c)L_1 - qH_G(y^*_c)L_2 \geq \Delta_F W
\]

by (18). Thus, if \(\Delta_F W = 0\), which means the two systems are indifferent to the judge under \(F\), the evidence-based system must be preferred under \(H\) since \(\Delta_H W > 0\).

Lemma 5. If \(\nu^{**}_I = 1\) and \(\nu^{**}_G > 0\), then \(\Delta_F W = 0\) implies that \(\Delta_H W > 0\).

Proof. First of all, when the innocent suspect is indifferent between confession or denial (i.e. along the line \(M_2\) in Figure 1), the social loss is the same as in Region \(III\), i.e. \(W^c = (1 - q)L_1\). The reason for this is that, if not, the judge can adjust the standard of proof such that the innocent suspect chooses the action that yields a lower social welfare loss. However, it contradicts that the innocent suspect is indifferent in the first place (so that it should not be on the line \(M_2\)).

Secondly, by taking the derivative of (13) with respect to \(\nu_G\), we have

\[
\frac{d\Delta_H W}{d\nu^*_G} = q\{H_G(x^*_c)L_2 + [1 - H_G(x^*_c)]kT\} > 0.
\]

Thus, we must have the result that \(\Delta_H W\) increases when a horizontal movement from Region \(III\) to Region \(II\) occurs. It means that when the suspects begin to deny the crime more likely, given \((L_1, L_2, k, T)\), torture performs worse and worse. This also explains why the line satisfying \(\Delta W = 0\) in Figure 2 has a positive slope in Region \(II\).
Figure 3. Change in Welfare when Information is Better

Hence, according to Lemma 4, and the fact that $\Delta_H W$ increases as $\nu^*_I$ and $\nu^*_G$ increase, if $\Delta_F W = 0$, then given parameters $(L_1, L_2, k, T)$, it must be that $\Delta_H W > 0$.

This is demonstrated in Figure 3, where the dot lines represent the outcomes under the better information. We can see that the line satisfying $\Delta W = 0$ shifts up when information becomes better, which means that torture is less efficient as information improves. It is also worthy mentioning that, as information becomes better, the suspects confess more likely when $\frac{L_2 - kT}{L_1 + kT}$ is relative small and less likely when $\frac{L_2 - kT}{L_1 + kT}$ is relative large. The reason is as follows. An equivalent way to say that information $H$ is better than $F$ is that signals are more correlated with the true states under $H$. In other words, the likelihood ratios become larger when $x$ is low and smaller when $x$ is high (Mathematically, $\frac{I}{I_G}$ becomes steeper). Since the smaller $\frac{L_2 - kT}{L_1 + kT}$, the higher $x^*_c$, the judge will choose a lower standard than before.
when information is more precise because high signals now more likely indicate that he is the guilty one. It follows that the probability to practice torture is higher than before. If the suspects always confess originally because of the fear of being tortured, he now has an even stronger incentive to confess. Therefore, the range of confession extends when \( \frac{L_2-kT}{L_1+kT} \) is relatively small. The analogous logic can be applied to the case when \( \frac{L_2-kT}{L_1+kT} \) is high.

This result has an important implication in the theory of judicial torture. When the investigation is not very precise (such as the Western society before the mid 18th century), torture can serve an efficient scheme in conviction as long as the cost by type I error is not serious, or the cost by type II error is very severe. However, as information becomes more and more precise, the judge can convict a suspect merely based on evidence instead of through torture. Our theory explains the process of the abolition of judicial torture starting from around mid-18th century.

5 Conclusion

This paper applies the economic theory of information to analyze the existence and abolition of judicial torture. It is shown that if the judge wishes to balance type I and type II errors in decision-making, and if during investigation very little information is revealed, then torture can maximize social welfare in that it forces the suspects to confess because of the fear of being tortured. However, as the information improves, the advantage of torture decreases. This may explain a transition from a confession-based system to an evidence-based system starting from the mid 18th century. Furthermore, torturing the witnesses might also be welfare-improving, as it helps to screen the cases so that only those with greater merits come into the court.
References


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