Partnership dissolution, complementarity, and investment incentives

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Abstract

Partnerships form in order to take advantage of complementary skills; however, new opportunities may arise that make some partners’ skills useless. We analyse partnerships that anticipate possible dissolution under the most commonly advised and widely used dissolution rule known as ‘buy-sell provision’. We find that this rule assures neither ex post efficient dissolutions nor ex ante efficient investments. We also discuss whether renegotiations, supplementing the buy–sell provision with the right to veto, or allowing the uninformed partner to set the dissolution price may restore efficiency, and whether preemptive requests for dissolution occur in equilibrium.

JEL classifications: D82, C78, J12, K12, L24.

1 Introduction

Partnerships are a frequently observed form of business organization among professionals. Examples range from lawfirms and medical practices, to engineering and business consulting. Typically, a partnership is an association of a few highly skilled professionals who have complementary skills and who are kept together by the promise of sharing the jointly earned profits and overhead expenditures. Indeed, roughly 80% of all partnerships have only two partners, and roughly two thirds of all two-partner partnerships exhibit equal share ownership (see Hauswald and Hege, 2003).

Partnerships form and dissolve. Frequently, one partner finds a new business opportunity which he may not want to share with his fellow partners, and therefore requests dissolution of the partnership. Other events that may trigger dissolution are offers from outsiders to purchase a partner’s share, divorce settlements in which a partner’s ex spouse receives a share in the firm, foreclosure of debt secured by a partner’s share, personal bankruptcy, and the disability or death of a partner, to name just a few.

Rational agents foresee that a partnership may unexpectedly be dissolved, no matter how promising it may be at the time when it is formed. Therefore, they take precautions and include a dissolution rule into the partnership contract, already at the time when the partnership is formed.

In business practice, the most commonly used dissolution rule is the ‘buy–sell provision’ (BSP) or ‘Texas shoot-out’, which is a variant of the well-known ‘I–cut–you–choose’ cake cutting mechanism. There, the partner who requests a dissolution
must propose a price at which the other partner may either sell his share or buy the proposer’s share. That rule is widely used in practice and recommended by legal advisors (see, for example, Mancuso and Laurence, 2003). Indeed, as Brooks and Spier (2004) report: “The importance of buy–sell agreements is now so broadly recognized that a lawyer’s failure to recommend or include them in modern joint venture agreements is considered ‘malpractice’ among legal scholars and practitioners.” Such mandatory provisions assure that a partnership is dissolved when it is called for and may avoid the cost of deadlocks, litigation, and lengthy court battles.

The present paper contributes to assess the efficiency properties of this common dissolution rule, by addressing the following questions:

(i) If dissolution is requested, does BSP ensure that ownership is awarded to the partner who values it most?

(ii) Does BSP ensure that dissolution is requested if and only if single ownership creates more value?

(iii) Does BSP ensure that the partnership chooses efficient investment, maximizing the present value of the sum of the partners’ gains?

The literature on partnership dissolution using BSP generally focuses on the first of these three efficiency issues. McAfee (1992) shows that BSP assures efficient assignment of single ownership under complete information. De Frutos and Kittsteiner (2008) extend this to two-sided incomplete information provided the right to propose dissolution is auctioned among partners. Brooks and Spier (2004) show that a mandatory buy–sell provision ensures that all potential gains from dissolution are realized when the partnership has common value and partners have asymmetric information.\(^1\)

One limitation of the partnership dissolution literature is that it takes the characteristics of the partnership and the dissolution decision as given and looks only at the issue of who shall be awarded single ownership. It thus ignores the question of whether the partnership should be dissolved at all, and how dissolution rules shape the very formation of partnerships and investment incentives (see Wolfstetter, 2002).

The anticipation of a possible dissolution affects the joint investment in the partnership, and in turn, the choice of investment affects the dissolution decision. For example, if partners expect a dissolution to occur with high probability, they may attempt to minimize the losses of the partner who withdraws from the firm in the event of a break-up, by choosing a relatively low investment, which in turn contributes to make such a break-up even more likely. Alternatively, partners may decide to go the other extreme, and overinvest into assets that increase the gains from complementarity to such an extent that a dissolution is effectively precluded. Some degree of such overinvestment is often observed in business, and is a conspicuous feature in many marital partnerships.

The present paper attempts to extend the partnership dissolution literature by analyzing the interrelationship between investment and the dissolution decision. For this purpose we introduce a simple, explicit partnership model, in which partnerships

\(^{1}\) Another branch of the literature examines the existence of a dissolution rule that assures an efficient assignment of single ownership under various information structures. See Cranton et al. (1987), Fieseler et al. (2003), Jehiel and Pauzner (2006), Minehart and Neeman (1999), and Ornelas and Turner (2007).
form in order to take advantage of complementary skills, as in Farrell and Scotchmer (1988). However, new opportunities may arise that make partners’ skills useless, and hence trigger a request for dissolution. Anticipating that possibility of a break-up, the partnership contract includes a dissolution rule which is typically a buy-sell provision. Investment in the partnership increases the gains from complementarity, yet makes potentially efficient dissolutions less likely.

Our main result is that BSP gives rise to inefficiency, either in the form of excessive dissolutions, combined with underinvestment, or in the form of efficient dissolutions, combined with overinvestment,\(^2\) although, in the present framework, it ensures the efficient assignment of single ownership conditional on dissolution. If the uninformed partner is given the right to set the dissolution price, full efficiency can be achieved in a subgame perfect equilibrium. This depends, however, on assigning full bargaining power to the uninformed partner. We then proceed to show that inefficiency is greatly reduced with a much simpler mechanism, BSP with a right to veto.

In the legal literature, the BSP involves a ‘partnership at will’ that shall be dissolved if at least one partner calls for dissolution. Whereas the BSP supplemented with the right to veto involves a ‘term partnership’ which no partner can lawfully leave without the consent of his fellow partners (see National Conference of Commissioners on Uniform State Laws, 1997). However, one has to keep in mind that a contract can always be breached. Therefore, even a term partnership can be dissolved, although at the risk of litigation for damages.

We also examine whether renegotiations may restore efficiency, and find that they are not a remedy. As in the case of adding a right to veto to BSP, renegotiations may achieve efficiency for some parameter sets, however, they give rise to an additional hold-up problem, allowing the informed partner to further exploit the uninformed.

The plan of the paper is as follows. In Section 2 the model is presented. In Section 3, we characterize the efficient partnership contract, consisting of joint investment and a dissolution rule. This result is then used as a benchmark to assess the BSP in Section 4. We show that it gives rise to either excessive dissolutions, combined with underinvestment, or efficient dissolutions, combined with overinvestment. That inefficiency prevails even if one allows a partner to call for a preemptive breakup, as shown in Section 5. In Section 6 we check what happens if the uninformed partner is given the right to set the dissolution price. In Section 7 we add the right to veto, and show that this may restore efficiency. In Section 8, we analyse how renegotiations affect the equilibrium. Section 9 concludes.

2 The model

Suppose two risk-neutral agents set up an equal share partnership in order to take advantage of their complementary skills. Before they pool their resources, they sign a partnership contract \(\{I, D\}\) that prescribes the joint investment \(I\) and the BSP dissolution rule, \(D\). Neither partner is capital constrained. Therefore, partners can agree ex

\(^2\)This result relates to the literature on cooperative and selfish investment (see, for example, Che and Chung, 1999, Che and Hausch, 1999, Rogerson, 1992). The driving force of that literature is the externality of investment, while ours is the buy–sell provision combined with one–sided asymmetric information.
ante on an investment policy that maximizes the total value of the partnership, without having to arrange side payments to account for unequal contributions to financing (which tend to occur if partners have different capital endowments).

After the partnership has been put in place, one randomly chosen partner finds a new business opportunity that may be incompatible with his partner’s skill. This triggers a reconsideration of the partnership which may lead to its dissolution.

The partnership is modeled as a sequential game, as follows:

In stage one, the two partners write the contract \( \{I,D\} \), set up the equal share partnership, and share the irreversible investment expenditure \( C(I) := \frac{1}{2}I^2 \).

In stage two, nature draws one of the partners with equal probability, and endows him with a new business opportunity. The partner who has received the new business opportunity is referred to as partner 1; the other as partner 2. Partner 1 may request a break-up, which is then executed according to the dissolution rule \( D \). Both partners know which role they have been assigned. And partner 2 is not allowed to call for a (preemptive) breakup (a possibility that is introduced later in Section 5).

The new opportunity gives rise to a profit shock \( \Pi \in \{0, \pi\} \), \( \pi > 0 \), which is either compatible or incompatible with the partners’ skills. It is described by three states of world, \( \Theta := \{th, nh, l\} \), which are drawn from the commonly known probability distribution \( q_s := \Pr\{S = s\}, s \in \Theta \). There, \( h, l \) indicate that the profit shock is either high, \( \Pi = \pi \), or low, \( \Pi = 0 \), and \( t, n \) (mnemonic for ‘team’ resp. ‘no team’) indicate that the innovation is compatible resp. incompatible with the partner’s skill. The realization of \( S \in \Theta \) is private information of partner 1.

If the partners stay together, each partner earns one half of the gross value of the firm, \( V_p(I, s) \):\(^3\)

\[
V_p(I, s) := \begin{cases} 
(1 + \alpha)I + \pi & \text{if } s = th \\
(1 + \alpha)I & \text{if } s \neq th 
\end{cases}
\]  
where \( \alpha > 0 \) is a measure of the complementarity of the partners’ skills.

Whereas, if the partnership is dissolved, the benefit of complementarity is lost, and the firm’s value is either \( V_1(I, s) \) or \( V_2(I, s) \), if partner 1, resp. partner 2, becomes single owner:

\[
V_1(I, s) := \begin{cases} 
I + \pi & \text{if } s \in \{th, nh\} \\
I & \text{if } s \in \{l\} 
\end{cases}
\]  
\[
V_2(I, s) := I, \text{ for all } s \in \Theta.
\]  

And the partner who exits has no benefit other than the transfer he receives in exchange for his ownership share.\(^4\)

Therefore, if partner 1 contemplates dissolution, he faces a trade-off between giving up the benefit of complementarity and pocketing his new business opportunity alone.

\(^3\)The qualitative results do not change in the alternative specification, \( V_p(I, th) = (1 + \alpha)(I + \pi) \), i.e., in which the complementarity applies also to the new opportunity.

\(^4\)This excludes that having entered into the partnership has created externalities such as knowledge spillovers which are common in high-tech partnerships.
The parameters \((\pi, \alpha)\) are constrained as follows:

\[
\pi \geq \alpha \left(1 + \alpha \left(1 - \frac{1}{2}q_{nh}\right)\right) =: \tilde{\pi}(\alpha) \quad (4)
\]

\[
\alpha \geq \frac{\sqrt{2} - 1}{1 - q_{nh} - q_{th}} =: \tilde{\alpha} \quad (5)
\]

and \(q_s : \Theta \to [0, 1], \sum_{s \in \Theta} q_s = 1\), has full support.

If (4) were not satisfied, it would never be efficient to break up. Constraint (5) is sufficient to ensure formation of the partnership if the partnership is dissolved only in either state \(nh\) or \(nh\) and \(th\). Partnership dissolution is a meaningful issue only if both constraints are satisfied (the detailed proof is contained in the downloadable version of the technical Appendix).

As an illustration of the profit shock \(\Pi\), suppose each partner has two skills, say \((a_1, b_1)\), and \((a_2, b_2)\); skills \(a_1\) and \(a_2\) are complementary, which is why the partnership forms in the first place, whereas skills \(b_1\) and \(b_2\) are used only alone. Then, state \(th\) would represent a positive demand shock for the products produced by combining the skills \(a_1\) and \(a_2\) and for the product produced with skill \(b_1\) alone, whereas state \(nh\) represents a positive demand shock for the goods produced with skill \(b_1\) alone.

The assumption that partners not only know the identity of the partner who has the new business opportunity, but can also exclude partner 2 from proposing dissolution, may seem to be critical. However, as we show in Section 5, the result is essentially unaffected if one allows partner 2 to call for a preemptive breakup.

The conclusions of the paper also remain unchanged if one assumes continuous instead of binary distribution of \(\Pi\) as long as that distribution is common knowledge.

3 Benchmark: Efficient investment and dissolution

At the outset we mention some basic facts about efficient dissolution. First, if the partnership is dissolved, single ownership should be awarded to partner 1, because \(V_1(I, s) \geq V_2(I, s)\). Second, dissolution is never efficient in states \(th\) and \(l\). Third, in state \(nh\) dissolution is efficient if and only if investment is lower than the level

\[
\tilde{I} := \pi / \alpha,
\]

because for \(I = \tilde{I}\) the loss of complementarity gain, \(\alpha I\), that results from dissolution, is equal to the gain from the new opportunity \(\pi\). This suggests that by choosing a sufficiently high investment (‘too-big-to-fail’ policy), the partnership can always prevent a future dissolution.

Let \(\mu_i(s) := \Pr\{\text{partner } i \text{ becomes single owner} \mid s\}, i \in \{1, 2\}\). As a benchmark, the first-best dissolution rule and investment are characterized in the following Lemma.

Lemma 1 The efficient dissolution rule, \(D^* := \{\mu_1^*(s), \mu_2^*(s)\}\), and joint investment, \(I^*\), are

\[
\mu_1^*(nh) = 1, \quad \mu_2^*(s) = 0, \forall s \neq nh, \quad \mu_1^*(s) = 0, \forall s \in \Theta \quad (7)
\]

\[
I^* := 1 + \alpha (1 - q_{nh}) < \tilde{I}. \quad (8)
\]
Proof From the above explanation it follows immediately that $\mu^*_{d}(s) = 0, \forall s$, and

$$
\mu_1(I, s) = \begin{cases} 
1 & \text{if } s = nh \text{ and } I \leq \tilde{I} \\
0 & \text{otherwise.} 
\end{cases}
$$

(9)

By assumption (4),

$$
\tilde{I} = \frac{\pi}{\alpha} \geq \frac{\tilde{\pi}(\alpha)}{\alpha} = 1 + \alpha \left(1 - \frac{1}{2} q_{nh}\right) > 1 + \alpha (1 - q_{nh}) = I^*.
$$

(10)

Therefore, (9) and (10) imply that $\mu_1(I^*, s) = \mu^*_{d}(s)$.

Finally, we confirm that the efficient investment level is indeed equal to $I^*$. Given the dissolution rule (9), the ex ante net value of the firm is

$$
V^*(I) := E_S \left[ \mu_1(I, S)V_1(I, S) + (1 - \mu_1(I, S)) V_p(I, S) \right] - C(I)
$$

$$
= \begin{cases} 
(1 + \alpha) I + q_{th} \pi - \frac{I^2}{2} =: \psi_1(I) & \text{if } I \geq \tilde{I} \\
(1 + \alpha) I + (q_{th} + q_{nh}) \pi - q_{nh} \alpha I - \frac{I^2}{2} =: \psi_2(I) & \text{if } I \leq \tilde{I}.
\end{cases}
$$

(11)

Due to (10), (4), and the strict concavity of $\psi_1, \psi_2$, one has

$$
I_1 := \arg \max_{I \geq \tilde{I}} \psi_1(I) = \max \{1 + \alpha, \tilde{I}\} \geq \tilde{I} > I^* = \arg \max_{I \leq \tilde{I}} \psi_2(I)
$$

$$
\psi_1(I_1) \leq \psi_1(1 + \alpha)
$$

$$
= \frac{(1 + \alpha)^2}{2} + q_{th} \pi + q_{nh} \left(\tilde{\pi}(\alpha) - \alpha \left(1 + \alpha \left(1 - \frac{1}{2} q_{nh}\right)\right)\right)
$$

$$
\leq \psi_2(I^*), \text{ since } \tilde{\pi}(\alpha) \leq \pi.
$$

Therefore, $I = I^*$ is the maximizer of $V^*(I)$, as asserted. □

We mention that the efficient allocation can be implemented by a call option, giving each partner the right to buy the other’s share at a particular strike price, if partners can verify the state of the world in the event when they stay together. However, the state of the world may not be verifiable. And it may be difficult to agree on a strike price, especially if, at the time when the partnership is formed, partners are subject to uncertainty concerning the state space and the relevant probability distribution. These may be some of the reasons why partnerships tend to prefer using a price-finding rule like the BSP that uses information available at the time of a dissolution instead of relying on the more fuzzy information available at the time of contracting.

4 Buy–sell provision

The partnership contract includes a buy–sell provision. If a partner calls for dissolution, he must quote a price; then, the other partner has the option to either sell his share or buy the proposer’s share at that stipulated price. We analyse the resulting partnership game, and find the perfect equilibrium partnership contract.

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5The use of such options has been proposed in a somewhat different context by Grossman and Hart (1986) and Nöldke and Schmidt (1998).
Following Samuelson (1984), we let the partner with more information be the proposer; this guarantees that the assignment of single ownership always maximizes the firm’s value. This assumes that the occurrence of the new business opportunity can be documented with hard evidence. Since this may not always apply, we will also explore what happens if there is no evidence and partner 2 may preemptively call for dissolution (see Section 5).

4.1 Dissolution subgame equilibrium

After the joint investment has been made, and the state of the world \( s \in \Theta \) has been realized and privately observed by partner 1, the two partners play the dissolution subgame. That game depends critically on the level of investment. As one would expect, if investment is very high, a dissolution is effectively precluded (‘too–big–to–fail’ policy), whereas a low level of investment gives rise to excessive dissolutions.

The strategies of partner 1 are denoted by \( \sigma_1(I,s) := (\tau_1(I,s), p(I,s)) \), where \( \tau_1(I,s) := \Pr\{\text{propose} \mid I, S = s \} \), and \( p(I,s) \) is the price quoted if a breakup is proposed in state \( s \). In turn, the strategy of partner 2 is his buy–sell decision, contingent upon the quoted price \( p \), denoted by \( \sigma_2(p) := \Pr\{\text{sell} \mid p\} \), where it is understood that \( 1 - \sigma_2(p) = \Pr\{\text{buy} \mid p\} \).

The solution of the dissolution subgame is explained in the following Lemmas.

Lemma 2 Partner 2 sells if and only if \( p \geq p^* := I/2 \), and partner 1 quotes the price \( p(s) = p^* \), if he proposes dissolution.

Proof Suppose partner 1 has proposed dissolution and quoted the price \( p \). If partner 2 buys, he earns the payoff \( V_2(I,s) - p = I - p \), whereas if he sells he earns \( p \). Therefore, he sells if and only if \( p \geq I/2 \). In turn, if partner 1 buys at price \( p^* \) he earns \( V_1(I,s) - p^* \geq I/2 \), whereas if he sells at price \( p < p^* \) he earns only \( p < I/2 \). Therefore, if he proposes, he quotes the price \( p = p^* \).

Lemma 2 says that when partner 1 proposes dissolution, he quotes a price that makes partner 2 indifferent between buying and selling. His information advantage allows him to extract the entire rent from dissolution. However, whether he indeed proposes dissolution depends on the level of investment.

Lemma 3 The dissolution subgame has the following equilibrium:

\[
\sigma_1(I,s) = (p^*, \tau_1(I,s)) \text{, } \tau_1(I,s) = \begin{cases} 1 & \text{if } s \in \Theta_1(I) \\ 0 & \text{otherwise} \end{cases}
\]

\[
\sigma_2(p) = \begin{cases} 1 & \text{if } p \geq p^* \\ 0 & \text{otherwise} \end{cases}
\]

where \( \Theta_1(I) = \begin{cases} \emptyset & \text{if } I \geq 2\tilde{I} \\ \{\text{nh}\} & \text{if } I \in [\tilde{I}, 2\tilde{I}) \\ \{\text{nh, th}\} & \text{if } I \in [0, \tilde{I}) \end{cases} \)
The equilibrium strategy $\sigma_2(p)$ and equilibrium price $p^*$ have already been established in Lemma 2. To confirm that $\tau_1(I,s)$ is part of partner 1’s equilibrium strategy, note that

$$
\frac{1}{2} V_P(I, th) \lesssim V_1(I, th) - \frac{I}{2} \iff I \lesssim \tilde{I}
$$

$$
\frac{1}{2} V_P(I, nh) \lesssim V_1(I, nh) - \frac{I}{2} \iff I \lesssim 2\tilde{I}
$$

Under the buy–sell provision the partnership is always dissolved when partner 1 proposes dissolution. Therefore, $\tilde{I}$ can be interpreted as the smallest investment that deters dissolution in state $s = th$, and $2\tilde{I}$ as the smallest investment that deters dissolution in all states. In other words, $I \geq 2\tilde{I}$ is a ‘too-big-to-fail’ policy. When investment is high, the opportunity cost of dissolving the partnership becomes high as the price partner 1 must quote in equilibrium is increasing in $I$.

### 4.2 Perfect Bayesian Nash equilibrium

To find the Perfect Bayesian Nash equilibria of the entire game, we compute the ex ante net value of the firm, for all choices of $I$, using the corresponding equilibrium of the above subgame (recall the definitions of $\psi_1$, $\psi_2$ in (11)):

$$
V(I) := E_S \left[ \tau_1(I, S) V_1(I, S) + (1 - \tau_1(I, S)) V_P(I, S) \right] - C(I)
$$

$$
= \begin{cases} 
\psi_1(I) & \text{if } I \geq 2\tilde{I} \\
\psi_2(I) & \text{if } I \in [\tilde{I}, 2\tilde{I}) \\
\psi_3(I) := \psi_2(I) - q_{th} \alpha I & \text{if } I \in [0, \tilde{I})
\end{cases}
$$

(15)

Let $\hat{I}$ be the maximizer of $\psi_3(I)$ on $[0, \tilde{I})$. In a first step we show that equilibrium investment is equal to either one of two levels, either $\hat{I}$ or $\tilde{I}$, which also implies that neither efficient investment nor the ‘too-big-to-fail’ investment level is part of the equilibrium.

**Lemma 4** The equilibrium investment is $I \in \{\hat{I}, \tilde{I}\}$, where

$$
\hat{I} = \arg \max_{I \in [0, \tilde{I})} \psi_3(I) = 1 + \alpha - (q_{nh} + q_{th}) \alpha < I^* < \tilde{I}.
$$

(16)

**Proof** The equilibrium investment is the maximizer of either $\psi_3$ on $[0, \tilde{I})$ or $\psi_2$ on $[\tilde{I}, 2\tilde{I})$ or $\psi_1$ on $[2\tilde{I}, +\infty)$. All three functions, $\psi_3$, $\psi_2$, $\psi_1$ are strictly concave.

First, note that $\hat{I}$ is the maximizer of $\psi_3$ on its domain, because $\psi'_3(\hat{I}) = 0$, and (using (10))

$$
\hat{I} = 1 + \alpha - (q_{nh} + q_{th}) \alpha < 1 + \alpha - q_{nh} \alpha = I^* < \tilde{I}.
$$

Second, $\tilde{I}$ is the maximizer of $\psi_2$ on its domain, since $\psi'_2(\tilde{I}) < 0$.

Third, $2\tilde{I}$ is the maximizer of $\psi_1$ on its domain, since $\psi'_1(2\tilde{I}) < 0$. 

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8
Finally, observe that:

\[ \psi_3(\hat{I}) - \psi_1(2\tilde{I}) = \frac{4\pi}{2\alpha^2} \left( \pi - \alpha (1 + \alpha - \frac{1}{2} q_{nh} \alpha) \right) + \frac{1}{2} (1 + q_t \alpha)^2 \geq \frac{1}{2} (1 + q_t \alpha)^2 > 0, \]  

by (4)

Therefore, \( I = 2\tilde{I} \) is dominated by \( I = \hat{I} \).

From Lemma 4 we see that the efficient investment \( I^* \) coincides with \( \hat{I} \) and is the optimal investment only if \( q_{th} = 0 \). Otherwise, the chosen investment level is equal to either \( \hat{I} \) or \( \tilde{I} \). If \( I^* \) is chosen, according to Lemma 3, the partnership will be dissolved in states \( th \) and \( nh \). Anticipating this, investing \( I^* \) is dominated by the lower investment \( \hat{I} \). However, if the complementarity between the two partners is large and \( \pi \) is relatively small, the partners may find it optimal to deter dissolution and exploit complementarity in state \( th \) by investing more than \( I^* \). Nevertheless, the 'too–big–to–fail' policy is never optimal because it makes the opportunity cost of dissolution so high that it completely precludes the chance of making use of a good outside option even in the state \( nh \).

**Lemma 5** The equilibrium investment is

\[ I = \hat{I} \iff (\pi, \alpha) \in \mathcal{P}_+ := \{(\pi, \alpha) \mid \pi \geq \max\{\pi_0(\alpha), \tilde{\pi}(\alpha)\}\} \]

\[ I = \tilde{I} \iff (\pi, \alpha) \in \mathcal{P}_- := \{(\pi, \alpha) \mid \tilde{\pi}(\alpha) \leq \pi < \max\{\pi_0(\alpha), \tilde{\pi}(\alpha)\}\}, \]

with \( \pi_0(\alpha) \) as defined in equation (17).

**Proof** To determine whether \( \hat{I} \) or \( \tilde{I} \) is optimal, compute the payoff difference

\[ \xi(\pi) := \psi_3(\hat{I}) - \psi_2(\tilde{I}) \]

\[ = \frac{1}{2\alpha^2} \left( \pi^2 - 2\pi \alpha (1 + (1 - q_{nh}) \alpha) + \alpha^2 (1 + (1 - q_{th} - q_{nh}) \alpha)^2 \right) \]

The following equation implicitly defines the set of parameters \((\pi, \alpha)\) for which \( \xi(\pi) = 0 \):

\[ \pi_0(\alpha) := \tilde{\pi}(\alpha) - \frac{1}{2} q_{nh} \alpha^2 + \sqrt{q_{th} \alpha^3 (2 + 2 \alpha - 2 q_{th} \alpha - q_{th} \alpha)}. \]  

(17)

Since \( \xi \) is increasing in \( \pi \) for all feasible parameters, \( \pi \geq \tilde{\pi}(\alpha) \), it follows immediately that \( \hat{I} \) is optimal if and only if \( \pi \geq \pi_0(\alpha) \) and \( \pi \geq \tilde{\pi}(\alpha) \). □

The two parameter sets, \( \mathcal{P}_+, \mathcal{P}_- \) are illustrated in Fig. 1. There, the area under the dotted curve and to the left of the vertical line \( \alpha = \tilde{\alpha} \) is the set of parameters that are not feasible (due to the constraints (4) and (5)), the area below the solid and at or above the dotted curve is the parameter set \( \mathcal{P}_- \), and the area at and above the solid curve is \( \mathcal{P}_+ \).

**Lemma 5** is intuitively appealing. For a given \( \alpha \), if \( \pi \) is sufficiently big, becoming a single owner is attractive to partner 1, which implies that the partnership is dissolved with high probability. This leads to a low level of investment \( (\hat{I} < I^*) \). In contrast, if \( \pi \)
is relatively small, retaining the partnership is more attractive to partner 1; as a result the equilibrium investment level is high ($\tilde{I} > I^*$).

Combining Lemmas 3–5, and recalling from Lemma 1 that efficiency means $I = I^*$ and dissolution if and only if $s = nh$, we conclude:

**Proposition 1**  
The perfect equilibrium exhibits:

(i) Excessive dissolutions (in $s \in \{nh, th\}$) and underinvestment, $I = \tilde{I} < I^*$, $\forall(\pi, \alpha) \in \mathcal{P}_+$.  

(ii) Efficient dissolutions and overinvestment, $I = \tilde{I} > I^*$, $\forall(\pi, \alpha) \in \mathcal{P}_-$.  

We close this subsection with two examples:

(i) Let $\alpha = 3$, $\pi = 16$, $q_{nh} = 0.5$, $q_{th} = 0.25$. This leads to excessive dissolutions, combined with underinvestment (illustrated in Fig. 2 on the left).

(ii) Let $\alpha = 3$, $\pi = 12$, $q_{nh} = 0.5$, $q_{th} = 0.25$. This leads to efficient dissolutions, combined with overinvestment (illustrated in Fig. 2 on the right).

The two examples differ only with regard to the magnitude of $\pi$. For the given $\alpha = 3$, the $\pi$ in the second example is relatively small; this makes it optimal for the partners to make an investment higher than $I^*$ in order to deter dissolution and exploit the complementarity in state $th$.

5 What if the uninformed partner may ‘preempt’?

We assumed so far that partners not only know the identity of the partner who has the new business opportunity, but can also exclude partner 2 from proposing dissolution. We now digress and show how the analysis changes if partner 2 cannot be excluded, for example because the evidence that indicates who has the new opportunity is not verifiable in court.

The main conclusion will be that the parameter set for which partnerships are feasible becomes more restrictive; however, if a partnership is feasible and generates higher
payoffs than going alone, the uninformed partner generally does not preempt and the solution of the game is as in Proposition 1.

The rules of the dissolution subgame are now specified as follows:

(i) Both partners simultaneously choose their dissolution strategy \((\tau_i, p_i)\).

(ii) If at least one partner proposes dissolution, the partnership is dissolved.

(iii) If both partners propose dissolution, i.e. if \(\tau_1 = \tau_2 = 1\), the higher price offer is declared the strike price of the buy-sell option, \(p = \max\{p_1, p_2\}\), and the partner who proposed the lower price is granted the buy-sell option; if they tie, the option is assigned at random.

(iv) If only one partner proposes dissolution, the other partner is granted the buy-sell option, as in the previous section.

Lemma 6 If partner 2 proposes dissolution, then \(p_2 = \frac{I + \pi}{2}\).

Proof Evidently, \(p_2 = \frac{I + \pi}{2}\) is the highest price at which partner 1 is ready to buy in the events \(s \in \{th, nh\}\). In order to induce partner 1 to sell, the price would have to exceed \(\frac{I + \pi}{2}\), and buying at such a price is less profitable for partner 2 than selling at \(p_2 = \frac{I + \pi}{2}\). \(\square\)

Lemma 7 If \(I \geq 2I\) (‘too-big-to-fail’ policy), then in equilibrium no partner will call for dissolution, i.e. \(\tau_1(s) = 0, \forall s\) and \(\tau_2 = 0\).

Proof We have already shown that it is not in the interest of partner 1 to call for dissolution in this case (see Lemma 3). Suppose partner 2 unilaterally calls for dissolution with \(p_2 = \frac{I + \pi}{2}\) (by Lemma 6). Call the associated payoff \(u_2(\tau_2 = 1)\) and the
payoff from not calling for dissolution $u_2(\tau_2 = 0)$. But then,

$$u_2(\tau_2 = 0) - u_2(\tau_2 = 1) \geq \frac{\pi}{2} (1 + q_{th} + 2q_l) > 0,$$

(18)

which is a contradiction. □

**Lemma 8** If, in equilibrium, either 1) $\tau_2 = 1$ and $I < 2\tilde{I}$, or 2) $I \geq 2\tilde{I}$, then going alone payoff dominates joining the partnership, in which case the partnership will not form.

**Proof** If $\tau_2 = 1$, the partnership dissolves with certainty. In this case it is obviously better to not join the partnership, because the investment cost is subadditive and thus joining the partnership does not pay if the complementarity benefit is never achieved.

$I > 2\tilde{I}$ is payoff dominated by $I = 2\tilde{I}$, and therefore does not occur in equilibrium.

If $I = 2\tilde{I}$, the partnership will never dissolve, by Lemma 7. Denote the expected gain from joining the partnership that chooses $I = 2\tilde{I}$ by $u_p$ and that from going alone and choosing the associated optimal investment (which is $I = 1$) by $u_a$, one finds:

$$u_a - u_p = \frac{1}{2} (1 + (q_{th} + q_{ih})\pi) - \frac{1}{2} \left( (1 + \alpha)2\tilde{I} + q_{th}\pi - \frac{1}{2}(2\tilde{I})^2 \right)$$

$$= \frac{2\pi^2 + \alpha^2 - 2\pi\alpha - 2\pi \alpha^2 + \pi q_{th}\alpha^2}{2\alpha^2}, \quad \text{by } \tilde{I} = \frac{\pi}{\alpha}$$

$$\geq \frac{2\pi^2 + \alpha^2 - 2\pi\alpha - 2\pi \alpha^2 + \pi (2\alpha(1+\alpha) - 2\pi)}{2\alpha^2}, \quad \text{by (4)}$$

$$= \frac{1}{2} > 0.$$

We conclude that going alone payoff dominates joining the partnership. □

**Proposition 2** If, in equilibrium, the partnership forms, partner 2 never calls for dissolution ($\tau_2 = 0$) and the equilibrium solution is the same as in the game where only partner 1 is permitted to propose. A sufficient condition is (4) combined with $q_l \geq 1/2$.

**Proof** From the above Lemmas it follows immediately that $\tau_2 = 0$ is a necessary condition for joining a partnership. In Appendix 1 we show in detail for which parameters $\tau_2 = 0$ is indeed part of the equilibrium. □

6 What if the uninformed partner sets the dissolution price?

In this section, we consider whether efficiency may be restored by separating the right of proposing dissolution and that of setting the dissolution price. For this purpose, we now consider the alternative specification of BSP where partner 2 sets the dissolution price, after partner 1 has proposed dissolution, and partner 1 can either accept or reject. The exact rules of the dissolution subgame are:6

---

6This alternative specification of BSP was suggested by an anonymous referee.
(i) Partner 1 proposes dissolution.\footnote{Similar to standard BSP, allowing partner 2 to propose dissolution preemptively generally does not change the equilibrium outcome.}

(ii) Partner 2 sets the dissolution price $p$.

(iii) Partner 1 decides whether to accept the dissolution price or not. If he rejects, the game ends and the partnership remains; if he accepts, the game proceeds to the next stage.

(iv) Partner 2 either buys partner 1’s share or sells his own share to partner 1 at the dissolution price $p$.

Similarly to equilibrium of the standard BSP, the equilibrium of the dissolution sub-game depends on $ex$ $ante$ investment.

Lemma 9 The dissolution subgame of the modified BSP has the following equilibrium: If $I \geq \tilde{I}$, partner 1 does not propose dissolution. If $I < \tilde{I}$, partner 1 proposes dissolution in all states. Partner 2 sets the dissolution price $p = \frac{1}{2}(1 - \alpha)I + \pi := \tilde{p}_2$ which is accepted by partner 1 if and only if the state is $nh$; partner 2 sells his share if partner 1 accepts the proposed dissolution price $\tilde{p}_2$.

The proof is in Appendix 2. By Lemma 9, the partnership is dissolved if $I < \tilde{I}$ and $s = nh$, which is $ex$ $post$ efficient. In the next proposition, we show that full efficiency is achieved in the subgame perfect equilibrium.

Proposition 3 If partner 2 sets the dissolution price, efficiency is implemented by the subgame perfect equilibrium.

Proof By Lemma 9, the partnership is dissolved if and only if $s = nh$ and $I < \tilde{I}$. When the partnership is dissolved, partner 1 becomes the single owner, which is $ex$ $post$ efficient. Given the efficient dissolution outcome, $I^*$ is the maximizer of the firm’s $ex$ $ante$ net value. 

Proposition 3 suggests that the inefficiency of the standard BSP is due to an uneven distribution of bargaining power; and that a redistribution of bargaining power through changing the game structure may improve efficiency.

7 Can the right to veto restore efficiency?

We now return to the setup where hard evidence of the new opportunity is available and consider the buy–sell provision modified by granting partner 2 the right to veto a proposed dissolution. This modification transforms the dissolution subgame into a signalling game in which the quoted price serves as a signal of partner 1’s private information, and partner 2 uses that signal to update his prior beliefs concerning the value of the partnership, in order to assess whether he should either sell his share or veto the dissolution and thus keep the partnership going.

In the following we employ the concept of a sequential equilibrium, characterized by strategies and beliefs that are consistent with those strategies. With slight abuse of
language, we will refer to the game played after a buy–sell provision has been offered as the dissolution ‘subgame’.

In the dissolution subgame with the right to veto, the action set of partner 2 has three elements: ‘buy’, ‘sell’, and ‘veto’. And partner 1 chooses between ‘propose’ a buy–sell provision and ‘don’t propose’. However,

Lemma 10 The dissolution subgame can be reduced to one where partner 1 always proposes and quotes a price $p \geq I_2$; and partner 2 only chooses between ‘sell’ and ‘veto’ (and never contemplates ‘buy’).

Proof (i) Observe that partner 2 will always veto, if partner 1 offers a price $p \in [I_2, I_2(1 + \alpha)]$, because veto gives him a payoff equal to $I_2(1 + \alpha)$ or more. Therefore, ‘don’t propose’ is payoff-equivalent to proposing a price $p \in [I_2, I_2(1 + \alpha)]$. We conclude that we can represent ‘don’t propose’ with ‘propose’ a price from that interval.

(ii) Observe that if partner 1 proposes a price $p < I_2$, partner 2 will either buy or veto, since buying is better than selling in that case. Instead of selling at such a price, partner 1 prefers to maintain the partnership. Therefore, in the light of 1), proposing a price $p < I_2$ is inferior to proposing a price $p \in [I_2, I_2(1 + \alpha)]$. We conclude that partner 1 will always propose and quote a price $p \geq I_2^8$.

7.1 A partial separating equilibrium that may restore efficiency

As in the game of BSP without the right to veto, the equilibrium of the dissolution subgame depends on the level of investment. Four intervals of investment must be distinguished:

$$I_1 := \left[0, \frac{q_{nh} \hat{I}}{2(q_{nh} + q_{sh})}\right], \quad I_2 := \left(\frac{q_{nh} \hat{I}}{2(q_{nh} + q_{sh})}, \frac{\hat{I}}{2}\right)$$

$$I_3 := \left[\frac{\hat{I}}{2}, \hat{I}\right], \quad I_4 := \left[\hat{I}, +\infty\right).$$

(19)

Lemma 11 The dissolution subgame has a ‘partial separating equilibrium’. There, dissolution occurs:

(i) never if $I$ is ‘high’: $I \in I_4$,
(ii) only in state $nh$ if $I \in I_3$,
(iii) in state $nh$ and with positive probability also in state $th$ if $I \in I_2$,
(iv) in states $nh$ and $th$ if $I$ is ‘low’: $I \in I_1$.

A detailed formulation and proof of Lemma 11 is in Appendix 3.

Notice that in such a separating equilibrium the critical investment required to deter dissolution becomes smaller than in the case without the right to veto. Now, the minimum investment required to deter dissolution in all states is equal to $\hat{I}$, and the minimum investment required to deter dissolution in state $th$ only is equal to $\frac{\hat{I}}{2}$.

We now show that, when partner 2 has the right to veto, the overall game has a perfect equilibrium, for some subset of the feasible parameters, that implements the efficient investment and dissolution rule.

8If $\alpha > 1$, this argument can be simplified, because in that case ‘veto’ dominates ‘buy’.
Proposition 4  The partial separating equilibrium implements the efficient investment and dissolution rule if and only if \( \pi \leq 2\tilde{\pi}(\alpha) - \alpha^2 q_{nh} \).

Proof  Suppose \( \pi \leq 2\tilde{\pi}(\alpha) - \alpha^2 q_{nh} \). For those parameters, one has \( I^* \in I_3 \). By definition of \( I^* \), the ex ante net value of the firm is maximized, provided dissolution occurs if and only if \( s = nh \). In the partial separating equilibrium, that condition is satisfied for all \( I \in I_3 \), and therefore, for \( I = I^* \).

Now suppose in equilibrium \( I = I^* \) and dissolution takes place if and only if \( s = nh \). Then \( \pi \leq 2\tilde{\pi}(\alpha) - \alpha^2 q_{nh} \) must hold. Suppose otherwise, i.e., \( \pi > 2\tilde{\pi}(\alpha) - \alpha^2 q_{nh} \). It follows that \( I^* < \frac{I}{2} \), which implies \( I^* \in \{I_1, I_2\} \). If the efficient investment level \( I^* \) is chosen, the partnership is dissolved in state \( nh \) and with a positive probability in state \( th \) by lemma 11, which is a contradiction.

Proposition 4 states a necessary and sufficient condition for an efficient equilibrium. If that condition is not satisfied, the partial separating equilibrium results in inefficiencies, in the sense that investment or dissolution efficiency are violated. Nevertheless, the buy-sell provision with veto right still performs better than the one without veto right since it leads to a higher expected firm value.

Corollary 1  Under the buy-sell provision with veto right the expected firm value is never lower than under the buy-sell provision without veto right.

When \( \pi \leq 2\tilde{\pi}(\alpha) - \alpha^2 q_{nh} \), the superiority of BSP with veto right is obvious. The proof of the remaining case is straightforward and hence contained only in the downloadable Appendix.

7.2 A partial pooling equilibrium that entails inefficiency

As in other signalling games, the dissolution subgame has also other equilibria, with different properties.

Lemma 12  The dissolution subgame has a ‘partial pooling equilibrium’. There, for all investment levels \( I \in I_3 \cup I_4 \) and all states \( s \) partner 1 quotes the same price at which partner 2 vetoes, and no dissolution occurs.

The detailed formulation and proof of Lemma 12 is also in the Appendix 3.

Proposition 5  The partial pooling equilibrium always entails inefficiency.

Proof  If \( I \in \{I_3, I_4\} \), by Lemma 12, in all states \( s \) partner 1 quotes the same price at which partner 2 vetoes; therefore, no dissolution occurs. If \( I \in \{I_1, I_2\} \), then the partnership is dissolved in states \( nh \) and with positive probability in state \( th \), which implies excessive dissolutions. Therefore, the outcome of the partial pooling equilibrium is always inefficient.

\footnote{Recall, the set of feasible \((\pi, \alpha)\) is \( \{(\pi, \alpha) | \pi \geq \tilde{\pi}(\alpha)\} \).}
As we have seen, adding the right to veto to BSP may restore efficiency for some parameters. However, it gives rise to multiple equilibria. If the inefficient equilibrium stated in Proposition 5 is played, under some parameters, dissolution is blocked when it is efficient. When partners expect that equilibrium to be played, they choose an investment level equal to $I_1$ which is higher than $I^*$ (‘too–big–to–fail’ policy). In such cases, both buy–sell provision with and without veto right are inefficient. No veto right ensures that a dissolution always occurs when it is called for while granting the veto right has the advantage of providing better investment incentives.

However, if one applies standard equilibrium refinements such as the ‘intuitive criterion’ introduced by Cho and Kreps (1987) and Banks and Sobel (1987), only the partial separating equilibrium in which partner 1 reveals his private information survives. The partial pooling equilibrium relies on partner 2’s off–equilibrium path belief that if partner 1 has quoted a very high price, the state of the world is believed to be $th$. That belief in turn is sustained by partner 1 playing a dominated strategy in state $nh$ (see the strategies and beliefs of Lemma 12 in Appendix 3). This suggests that one can assert with confidence that adding veto power tends to restore efficiency.

Although efficiency can also be restored by granting full bargaining power to partner 2, as shown in section 6, implementation of bargaining power allocation is generally difficult and may rely on complex sets of fines and unrealistic behavioral assumptions about how agents solve long-chains of backward induction problems (see Aghion et al., 1994). In contrast, BSP with veto is very simple and easy to understand, and it does not require any complex multistage game specified legally or high order common knowledge of rationality.

8 Can renegotiations restore efficiency?

Can partners restore efficiency through renegotiations? The presence of inefficient dissolutions gives the partners an incentive to renege the BSP rule. However, as in other asymmetric information problems, renegotiations can only restore efficiency in some cases (see, for example, Matthews, 1995).

After partner 1 has requested dissolution one partner may propose to renege the default BSP agreement in order to keep the partnership together in exchange for some transfer. This may then trigger a bargaining process. For simplicity, we assume the simplest possible bargaining process, where one party makes an ultimatum offer. If that offer is accepted, the partnership is maintained and a stipulated transfer is paid. Otherwise it is dissolved according to the default BSP agreement at the strike price initially proposed by partner 1. Two cases must be distinguished: (i) the informed party, partner 1, makes the renegotiation offer, and (ii) partner 2 takes the initiative. The first case gives rise to a signalling and the second to a screening problem.

8.1 Signalling

In the signalling case, for $I \in \Lambda := \left\{ \frac{q_l I}{2q_h + q_l}, \frac{(q_l + 2q_h)I}{2(q_h + q_l)} \right\} \subset [0, \check{I}]$, the dissolution/ renegotiation subgame has a partial separating equilibrium in which the partnership is only dissolved in state $nh$. In that subgame equilibrium, partner 1 proposes dissolution in all states and offers to renegotiate in states $\{th, l\}$ by asking for a positive transfer $t = \frac{q_l I}{2} + \frac{q_h \pi}{2q_h + q_l}$ from partner 2. The renegotiation proposal by partner 1 perfectly
differentiates state \( nh \) from other states, and partner 2 takes the signal and accepts the renegotiation offer when it is proposed. One can further show that if the parameters satisfy \( \pi(\alpha) \in \left[ \frac{2(q_l + q_{nh})}{q_l + 2q_{nh}}, \left( \frac{q_{nh} + q_l}{q_l} \right) \left( 2\tilde{\pi}(\alpha) - q_{nh}\alpha^2 \right) \right] \), the above subgame equilibrium forms a perfect equilibrium of the entire game and the equilibrium investment is \( I = I^* \), which implies that efficiency is achieved.

However, as in other signalling games, the dissolution/renegotiation subgame has also pooling equilibria for some parameters which, by definition, cannot implement efficiency. Moreover, as the above equilibrium illustrates, renegotiation creates an additional hold-up problem by granting partner 1 more opportunity to extract surplus from the weaker partner.

### 8.2 Screening

Instead of letting the informed party propose renegotiation one can, of course, let the uninformed partner 2 be the proposer. Interestingly, in this case one finds a pooling equilibrium that implements full efficiency for a small set of parameters. That equilibrium is characterized as follows.

The partnership chooses the efficient investment \( I = I^* \). Partner 1 proposes dissolution in all states with price \( p = p^* = I/2 \), partner 2 renegotiates and offers the transfer \( \bar{t} = (\pi - \alpha I)/2 \) in exchange for keeping the partnership, and partner 1 accepts iff \( t \in [\tilde{t}_+, \tilde{t}_-] \) and \( s \in \{ lh, l \} \), or \( t \in [\tilde{t}_-, \tilde{t}_+] \) and \( s = l \) or in all states if \( t \geq \tilde{t}_+ \), where \( \tilde{t}_+ \) denotes the smallest transfer that ensures acceptance in all states and \( \tilde{t}_- \) the highest transfer that ensures acceptance in state \( l \). The beliefs of partner 2 are such that priors are confirmed if dissolution is requested. One can show that there exists a small parameter set for which these strategies are mutual best replies and the beliefs are consistent with them. As a result, the partnership is dissolved only in state \( s = nh \), which is efficient for the given investment \( I^* \).

However, the above equilibrium survives only for a fairly small parameter set. For example, it fails to exist if \( q_l + q_{nh} > 1/2 \) or if \( q_{nh} > q_{lh} \). Moreover, it gives rise to the same hold-up problem as when partner 1 is given the option of requesting for renegotiation.

### 9 Conclusion

With this paper we have extended the partnership dissolution literature, initiated by Cramton et al. (1987), by setting up an explicit model of partnerships that may explain why partnerships form and yet dissolve in the face of new business opportunities. We analysed the effect of the commonly advised and frequently used dissolution rule, known as buy–sell provision. That rule ensures that, conditional on dissolution, the assignment of single ownership is efficient. However, it always entails an efficiency loss, either in the form of excessive dissolutions, combined with underinvestment, or efficient dissolutions, combined with overinvestment. When a right to veto is added, efficiency may be restored. When the uninformed partner has the right of setting the dissolution price, full efficiency is achieved. When renegotiation is allowed, efficiency may be restored in some cases; however, it gives rise to an additional hold-up problem where the informed partner extracts rent from the uninformed partner.
One testable hypothesis is that the buy–sell provision with a veto right provides better investment incentives. Therefore, when it is important to protect specific investments, one might expect to see more term partnerships which partners cannot leave without the consent of their fellow partners (or one has to compensate for the damages in case of breach of contract).

Finally, although our model focuses on buy–sell provision, it should be regarded as a starting point for a more general analysis of the interaction between investment and dissolution incentives. For example, it would be interesting to see what happens if investments are intangible services that may not be contractible. We conjecture that this makes the inefficiency problem caused by BSP more severe, since it gives rise to an additional free riding problem. It would also be interesting to see how other dissolution rules perform, such as winner’s bid auction, loser’s bid auction, and bargaining with alternating offers.

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Appendix

Here we spell out the proofs that were not included in the main text. An extended version that fully covers all cases is available for download at http://www2.wiwi.hu-berlin.de/institute/wt1/research/2009/extended-appendix.pdf.

Appendix 1 Proof of Proposition 2

Here we solve the dissolution subgames assuming that both partners may call for dissolution (as in Section 5) and prove the sufficient condition stated in Proposition 2.

We have already shown in Lemmas 6–8 that if partner 2 proposes, he proposes the price \( p_2 = \frac{I + \pi}{2} \); furthermore, in equilibrium \( I < 2 \hat{I} \). Therefore, we only need to solve the dissolution subgames for (i) \( I \in [0, \hat{I}) \), (ii) \( I \in [\hat{I}, 2 \hat{I}) \).

Lemma 13 Suppose \( I < \hat{I} \). Then, the equilibrium dissolution strategies are: Partner 1 calls for dissolution and sets \( p_1 = \frac{I}{2} \), if and only if \( s \in \{th, nh\} \); and if he gets the buy-sell option, ‘buys’ if and only if the strike price \( p \leq \frac{I + \pi}{2} \) and \( s \in \{th, nh\} \). Partner 2 calls for dissolution, i.e. \( \tau_2(I) = 1 \), sets \( p_2 = \frac{I + \pi}{2} \), if and only if \( (q_t, I) \in S_1 := \{(q_t, I) \mid q_t \leq \frac{\pi}{2r + a} \} \); if he gets the buy-sell option, he sells if and only if the strike price is \( p \geq \frac{I}{2} \).
Proof Suppose partner 1 plays the asserted equilibrium strategy. We determine for which parameters it is a best reply of partner 2 to play \( \tau_2(I) = 1, p_2 = \frac{1+\pi}{2} \).

Denote the payoff of partner 2 if he plays \( \tau_2(I) = 1 \) by \( u_2 \) and that if he plays \( \tau_2(I) = 0 \) by \( u_2' \). Then,

\[
u_2 - u_2' = \frac{I + \pi}{2}(1 - 2q_l) + q_l I - \left(1 - q_l\right)\frac{I}{2} + q_l (1 + \alpha) \frac{I}{2} \geq 0 \iff (q_l, I) \in S_1.
\]

Next, suppose partner 2 plays the asserted equilibrium strategy. If that strategy prescribes \( \tau_2(I) = 0 \), we are back in the game where only partner 1 proposes (see Lemma 3). If \( \tau_2(I) = 1 \), any price below \( p_2 \) (including the above stated price \( p_1 \)) is a best reply of partner 1. Because then partner 1 buys at \( p_2 \) if and only if \( s \in \{th, nh\} \) and thus earns the payoff \( u_1 = \frac{I + \pi}{2} \), regardless of which state occurred; whereas if he quotes a higher price than \( p_2 \), partner 2 will sell, which will lead to the payoff

\[
u_1' = \begin{cases} I + \pi - p_1 & \text{if } s \in \{th, nh\} \\ I - p_1 & \text{otherwise,} \end{cases}
\]

which is obviously smaller than \( u_1 \). Hence, the asserted strategies are mutual best replies. □

Lemma 14 Suppose \( I \in [\tilde{I}, 2\tilde{I}) \). Then, the equilibrium dissolution strategies are: Partner 1 calls for dissolution and sets \( p_1 = \frac{I}{2} \) if and only if \( s = nh \); and if he gets the buy-sell option, ‘buys’ if and only if the strike price \( p \leq \frac{I + \pi}{2} \) and \( s \in \{th, nh\} \). Partner 2 calls for dissolution, i.e. \( \tau_2(I) = 1 \) and \( p_2 = \frac{I + \pi}{2} \), if and only if \( (q_l, q_{th}, I) \in S_2 := \{(q_l, q_{th}, I) \mid q_l \leq \frac{\pi(1-q_{th}-\alpha I)q_{th}}{2\pi+\alpha I q_{th}}\} \); if he gets the buy-sell option, he sells if and only if the strike price is \( p \geq \frac{I}{2} \).

Proof The proof is similar to the proof of Lemma 13 and hence omitted. □

From these two Lemmas we conclude that in equilibrium partner 2 never calls for dissolution if \( q_l \geq \frac{1}{2} \), as asserted in Proposition 2.

Appendix 2 Proof of Lemma 9

Proof If \( I < \tilde{I} \), at stage 2, partner 2 has a weakly dominant strategy of setting the dissolution price at \( p = \tilde{p}_2 = \frac{1}{2}(1-\alpha)I + \pi \), which is accepted by partner 1 if and only if \( s = nh \), and partner 2 sells his share when partner 1 accepts. A higher price than \( \tilde{p}_2 \) does not change anything if \( s \in \{th, I\} \), but changes partner 1’s decision at stage 3 from ‘accept’ to ‘reject’ if \( s = nh \). In that case partner 2’s payoff is reduced by

\[
\frac{1}{2}(1-\alpha)I + \pi - \frac{1}{2}(1+\alpha)I = \pi - \alpha I > 0
\]

Any price lower than \( \tilde{p}_2 \) but above \( p^* = I/2 \) reduces partner 2’s payoff in state \( nh \) as partner 2 sells his share at a cheaper price. If the price is reduced to \( \tilde{p}_2 - \frac{\pi}{2} \) or lower,
it also changes partner 1’s decision in state th from ‘reject’ to ‘accept’, this, however, reduces partner 2’s payoff by

\[
\frac{1}{2} ((1 + \alpha) I + \pi) - \left( \tilde{p}_2 - \frac{1}{2} \pi \right) = \alpha I > 0 \tag{21}
\]

If the price is reduced to a level lower than \( p^* \), partner 2’s best strategy at the final stage becomes ‘buy’, and partner 1’s best response at stage 3 is to ‘reject’ for any \( s \). As a result, partner 2 receives a lower payoff than by setting the price at \( p = \tilde{p}_2 \), as shown above for the case \( p > \tilde{p}_2 \). Therefore, setting a price other than \( p = \tilde{p}_2 \) always reduces partner 2’s payoff if it changes anything. Given partner 2’s strategy, it is a best response for partner 1 to propose dissolution in any state at stage 1.

If \( I \geq \tilde{I} \), setting some price \( p > \tilde{p}_2 \) is a best choice for partner 2. That price is rejected by partner 1 in all states. A price lower than \( \tilde{p}_2 \) changes partner 1’s decision at stage 3 from ‘reject’ to ‘accept’ if \( s = nh \) and partner 2’s payoff is reduced by at least

\[
\frac{1}{2} (1 + \alpha) I - \left( \frac{1}{2} (1 - \alpha) I + \pi \right) = \alpha I - \pi > 0 \tag{22}
\]

Lowering the price further to change partner 1’s decision from ‘reject’ to ‘accept’ in state \( th \) also further reduces partner 2’s expected payoff. A price below \( I/2 \) will be rejected by partner 1 in all states as partner 2’s best strategy at the final stage is to ‘buy’ rather than ‘sell’. Hence, a dissolution price below \( I/2 \) is no improvement over \( p > \tilde{p}_2 \) for partner 2. Therefore, setting a price above \( \tilde{p}_2 \) is a best choice for partner 2 when \( I \geq \tilde{I} \). Given partner 2’s strategy, it is a best response for partner 1 not to propose any dissolution at stage 1. \( \square \)

### Appendix 3 Proof of Lemmas 11, 12

In the reduced game under BSP with the right to veto, the strategy of partner 1 is his probability of quoting a price \( p \), denoted by \( \sigma_1(p; I, s) := \Pr\{P = p \mid S = s\} \), with some support \( \mathcal{P} \). The strategy of partner 2 is \( \sigma_2(p; I) = \Pr\{\text{sell} \mid p\} \) and \( 1 - \sigma_2(p; I) = \Pr\{\text{veto} \mid p\} \). And the beliefs of partner 2 are denoted by \( \delta_s(p, I) := \Pr\{S = s \mid p\} \).

Here we give detailed statements and proofs of Lemmas 11 and 12.

**Lemma 11** The equilibrium strategies and beliefs of the ‘partial separating equilibrium’ are:
Strategies:

\[ \sigma_1(\hat{p}(I); th, I) := \eta(I), \quad \sigma_1(p'_1; th, I) := 1 - \eta(I) \]  
\[ \sigma_1(\hat{p}(I); nh, I) := 1, \quad \sigma_1(p'_1; l, I) := 1 \]  
\[ \frac{I}{2} \leq p_1 < p'_1 < \frac{(1 + \alpha)I}{2} \leq \hat{p}(I) := \frac{1}{2} \left( I(1 + \alpha) + \delta_{th}(\hat{p}(I), I)\pi \right) \]  
\[ \sigma_2(p; I) = \begin{cases} 
1 & \text{if } p > \hat{p}(I) \text{ or } \left( p = \hat{p}(I) \text{ and } I < \tilde{I} \right) \\
0 & \text{otherwise} 
\end{cases} \]  
\[ \eta(I) := \begin{cases} 
0 & \text{if } I \in I_3 \cup I_4 \\
\frac{q_{nh}(\pi - 2\alpha I)}{2q_{nh}I} & \text{if } I \in I_2 \\
1 & \text{if } I \in I_1 
\end{cases} \]  

Beliefs:

\[ \delta_{th}(p, I) := \begin{cases} 
1 & \text{if } p \in [p'_1, \hat{p}(I)) \\
\frac{q_{th}\sigma_1(\hat{p}(I); th, I)}{q_{th} + q_{th}\sigma_1(\hat{p}(I); th, I)} & \text{if } p = \hat{p}(I) \\
0 & \text{if } p < p'_1 \text{ or } p > \hat{p}(I) 
\end{cases} \]  
\[ \delta_{nh}(p, I) := \begin{cases} 
1 & \text{if } p > \hat{p}(I) \\
\frac{q_{nh}}{q_{nh} + q_{nh}\sigma_1(\hat{p}(I); th, I)} & \text{if } p = \hat{p}(I) \\
0 & \text{if } p < \hat{p}(I) 
\end{cases} \]  
\[ \delta_l(p, I) := \begin{cases} 
1 & \text{if } p < p'_1 \\
0 & \text{otherwise} 
\end{cases} \]  

Proof The beliefs are obviously consistent with the stated strategies, using Bayes’ rule, when it applies. Also, partner 2’s strategy is evidently a best reply, given his beliefs. It remains to be shown that partner 1’s strategies are best replies, given the beliefs \( \delta(p, I) \), for all investment levels.

The proof requires comparing partner 1’s payoff when he follows the prescribed strategy and when he deviates. This tedious and mechanical exercise is available in the downloadable extended version of the appendix.

Lemma 12 The equilibrium strategies and beliefs of the ‘partial pooling equilibrium’ for \( I \in I_3 \cup I_4 \) are:

Strategies:

\[ \sigma_1(p_1; s, I) := 1, \text{ for all } s \in \Theta \]  
\[ \frac{I}{2} \leq p_1 < \hat{p}(I) := \frac{1}{2} \left( I(1 + \alpha) + \pi \right) \]  
\[ \sigma_2(p; I) = \begin{cases} 
1 & \text{if } p \geq \hat{p}(I) \\
0 & \text{otherwise} 
\end{cases} \]
Beliefs:

\[
\delta_{th}(p, I) := \begin{cases} 
1 & \text{if } p \geq \hat{p}(I) \\
q_{th} & \text{if } p \in [p_1, \hat{p}(I)) \\
0 & \text{otherwise}
\end{cases} \quad (34)
\]

\[
\delta_{nh}(p, I) := \begin{cases} 
q_{nh} & \text{if } p \in [p_1, \hat{p}(I)) \\
0 & \text{otherwise}
\end{cases} \quad (35)
\]

\[
\delta_l(p, I) := \begin{cases} 
q_l & \text{if } p \in [p_1, \hat{p}(I)) \\
1 & \text{if } p < p_1 \\
0 & \text{otherwise}
\end{cases} \quad (36)
\]

**Proof** The beliefs are obviously consistent with the stated strategies, using Bayes’ rule, when it applies. Also, partner 2’s strategy is evidently a best reply, given his beliefs. Partner 1 can only make a difference if he deviates and quotes a price \( p \geq \hat{p}(I) \), at which partner 2 sells for sure. However, that never pays. □

**References**


