The Case of the Internet Twins as a Generalization of King Solomon's Dilemma *

by

Jin Kim

Korea Institute of Public Finance
79-6 Garak-Dong, Songpa-Gu
Seoul, 138-774, Korea
jkim@kipf.re.kr
Telephone: 82-2-2186-2220
Fax: 82-2-2186-2309

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Abstract

Motivated by a recent international custody battle over the Internet Twins, we study a generalized problem of King Solomon's dilemma in that nobody knows who has the highest valuation for the baby when valuations are private information. We propose as optimal mechanism a coordination system of law firms with monetary transfers to assign the baby to the client who has the highest valuation. Our coordination system also incorporates the absolute truth clause in testimony, voluntary participation, and acceptable bargaining among member companies. Our finding can be generally applied into privatization problems with incomplete information.

Key Words: King Solomon's Dilemma, Internet Twins, Adoption Litigation, Privatization, Dominant Strategy Mechanisms

JEL numbers: C72, D44, D61, D74, D82

1. Introduction

King Solomon's judgment in the Bible has recently been one of the main examples used in the mechanism design literature. (1) Solomon was able to find the genuine mother of a baby through a litigation process. Perry et al. (1999) solved this baby award problem through a finite-stage mechanism by assuming that it is common knowledge (2) that the genuine mother of the baby knows that it is her baby. But what would King Solomon do today when, it can be said, neither woman knows who the genuine mother is?

Quite a surprising example would be a recent international custody battle over the Internet Twins: Two sets of parents, one English couple and one California couple, had paid to adopt twin babies, born in the U.S., via a Web-site called the Caring Heart Adoption. Though two girls, Kimberley and Belinda, are now in the U.S. foster care because their estranged biological parents sought to regain custody respectively, which couple of parents would have got the twins if the courts in the U.K. and U.S. had had to rule this case? Mrs. Allen of the California couple seems to say one possible answer, " I have an interest ... with their being placed with the right parents." (3) Then which would have been the 'right' parents of the internet twins? Finding the right owner of an indivisible private good (the prize) is the typical questions faced everyday by central agencies who carry out modern privatization all over the world; from the privatization of radio frequency bands in the U.S. to the privatization of national facilties in Eastern Europe.

In this paper, we study the finding of the right owner by considering a pre-trial negotiation system of law companies with monetary transfers in a setup of mechanism design. Formally, we examine the possibility of a mechanism through which the law firms with monetary transfer schemes assigns an indivisible object (babies) to the client with the highest valuation. We also impose the following properties on the pre-trial coordination system: First, a mechanism should promote the absolutely honest testimony in the process; the absolute truthness. Second, a mechanism should rule any case substantially without enforcement; voluntary participation. Finally, a mechanism should in expection guarantee

(1) Bag (1996), Moore (1992), and Glazer et al. (1989). See also Binmore (1998).
(2) See Aumann (1976) for a formal concept.
(3) See http://www8.cnn.com/2001/WORLD/europe/04/10/internet.twins/index.html.

the potential gain of each firm; acceptability.

Our model uses two properties of modern litigation systems; a diversified client base and bargaining practices among law firms in the litigation industry. As a member of the litigation industry, a practitioner firm represents a pool of various potential clients. We explicitely incorporate the bargaining among law companies in the litigation industry in that the companies may use monetary transfers in coordination to rule the case. Then the question is how to establish a reliable institutional arrangement with incentive transfer schemes.

The remainder of this paper is organized as follows. In section 2, the model and basic terms are presented. Section 3 has our main result, the existence of a coordination system with monetary transfers that satisfies the above three institutional constraints. In section 4, a two-agent example is displayed as a baby adoption problem.

2. The Model

There is a finite set of law companies (agents), $I = \{1, 2, \cdots, n\}$, with a typical element $i \in I$. Each firm expects to have a pool of clients and is ready to fight for winning cases on the behalf of its clients. A case happens when two (or several) [4] persons disputes on the ownership of the prize and want to hire law companies respectively to win the prize. As a case happens and the pairs of client-lawyer are created, each pair shares information on the valuation of its client completely. The valuations of clients are formulated as company's types. We assume that the distribution of law firms' types are public information. Let $\Theta_i = [b_i, c_i]$ be a set of possible types of clients for company $i \in I$ with $c_i > b_i \geq 0$. Then the set of cases (or states) is $\Theta \equiv \prod_{i=1}^{n} \Theta_i$ with a typical element $\theta = (\theta_1, \cdots, \theta_n)$. A common prior (cumulative distribution) on Θ is given by a probability measure F on the (Borel) subsets of Θ.

We assume that law companies want to establish an ex-ante arrangement to rule cases. There are several methods of assignment decision but eventually all the methods propose a distribution of property right ratios among the lawyer-client pairs. For example, a random

(4) Each prize will be acclaimed by a subset of I.

assignment through a lot is one method to assign the ownership of the prize, where the probability of winning the prize is $\frac{1}{n}$ to each lawyer-client pair. Let A denote the set of outcomes with a typical element $a = (a_1, \cdots, a_n)$ where $A = \{(a_1, \cdots, a_n) \in \Re^n | \, 1 \geq a_i \geq 0$ for all i and $\sum_i a_i = 1\}$. There might be a method with more formal structure than the random method. The most formal method is the court system. In this paper we propose a pre-trial coordination system among law firms. Formally, law firms want to build a coordination system to rule the ownership of an indivisible private good (the baby). After installing ex ante arrangement such as a coordination system, for each case it decides who owns the prize.

We assume that clients and law firms have quasi-linear preferences on the pairs of outcome and money. Thus, we assume that a coordination system can use monetary transfers among law companies. Specifically, the payoff of client with θ_i of company i from outcome $a \in A$ and monetary transfer $m_i \in \Re$ is $a_i\theta_i + m_i$. The payoff of company i is then $a_i v(\theta_i) + m_i$ where $v(\cdot)$ is a continuous and strictly increasing function such that $\theta_i \geq v(\theta_i)$ for all i and for all θ_i. We may interpret $v(\cdot)$ as a common reward scheme to lawyers. [5] Thus, client with θ_i has a valuation $\theta_i - v(\theta_i)$ for winning.

Socially optimal decision would be giving the baby to the client with the highest valuation. This judgment seems to be 'just' in the sense that the 'right' parent obtains the custody of the baby. We assume that the society exerts to promote this social justice. Formally, define the **social gain function** $g(\cdot)$ as $g(\theta) = \theta^* = \max_j \theta_j$ for each θ. If the society can keep the social gain function, the client with the highest valuation receives the baby at all cases. The objective of the law firms with cooperative interest would be to realize

$$v(g(\theta)) \equiv v(\theta^*) = v(\max_j \theta_j). \tag{1}$$

Then the social gain function g results in the winning states for each agent i as $W^i = \{\theta \mid \theta_i = \theta^*\}$. Then, we can define a potential gain for company i as the average of i's payoffs from the social decision,

$$G_i \equiv E[\mathbf{1}_{W^i}(\theta) v(\theta_i)] \tag{2}$$

(5) Hendricks et al. (2000) has a common value function in a merger analysis.

where E is the expectation operator with respect to F and $\mathbf{1}_{W^i}(\theta)$ is an index function such that its value is 1 if $\theta \in W^i$, or 0 otherwise.

A coordination system wants to achieve $g(\theta)$ in (1) for all θ. But, due to the incompleteness of information, the system needs to use a monetary incentive scheme to induce the revelation of information from law companies. We want to use a game form to construct an incentive compatible coordination system. A game form or mechanism, which consists of a message space and an outcome function, should be constructed to give each company an incentive to display the truthful testimony on the behalf of clients. This incentive compatibility makes us restrict our focus on direct mechanisms: Due to Revelation Principle [(6)], we have no loss of generality in restricting our attention to direct mechanisms.

A direct mechanism is denoted by $(\Theta, < s, t >)$ where Θ is the message space of type reports and $< s, t >$ is an outcome function which consists of an assigning rule $s : \Theta \rightarrow A$ and a transfer scheme $t : \Theta \rightarrow \Re$. Given a message $\theta' \in \Theta$, $s(\theta') = (s_1(\theta'), \cdots, s_n(\theta'))$ assigns property right ratios and $t(\theta') = (t_1(\theta'), \cdots, t_n(\theta'))$ designates transfers. I,e., $s_i(\theta')$ is the property right ratio of firm i's client with θ_i on the baby and $t_i(\theta')$ is the transfer from the coordination system to firm i when firms' reports are θ'. Given $< s, t >$, company i's payoff with client's type θ_i and reports θ' is $s_i(\theta')v(\theta_i) + t_i(\theta')$. We will abuse the notation $< s, t >$ as a direct mechanism and call it a (direct) coordination system.

Once a coordination of law companies is installed, they face a direct revelation game with incomplete information after each company knows the valuation of its client. We will use dominant strategy equilibria as our equilibrium concept. Only for the comparison with Bayesian equilibria, we use the assumption that companies' types are independent, i.e., $F = \prod_{i=1}^{n} F_i$ and $F_{-i} = \prod_{j \neq i} F_j$ where F_i is the marginal distribution of F on Θ_i.

A coordination system $< s, t >$ is absolutely truthful (AT) if every law company has an incentive to report the type of its client honestly in a direct revelation game regardless of others' report schemes, i.e., for all i, for all θ_{-i}, for all θ_i, and for all θ_i',

$$s_i(\theta_{-i}, \theta_i)v(\theta_i) + t_i(\theta_{-i}, \theta_i) \geq s_i(\theta_{-i}, \theta_i')v(\theta_i) + t_i(\theta_{-i}, \theta_i'). \tag{3}$$

This is the same as the dominant-strategy incentive compatibility in the mechanism design

[(6)] See Dasgupta et al. (1978) and Hurwicz et al. (1985).

literature. This is the so-called 'absolute truth' or 'nothing-but-the-truth' clause in the courts.

A coordination system $< s, t >$ is first-best if it realizes the social gain function g in any case. In a first-best coordination system, the client with the highest valuation obtains the baby in each case. Furthermore, each law company wins the cases in its winning states and gets its potential gain in (2) directly from the assignment of the system regardless of transfers. Note that the assignment rule s in a first-best coordination system is unique.

A coordination system $< s, t >$ is ex post individually rational (EPIR) if for each case every company has no incentive to drop out from the court ruling after knowing the whole case, i.e., for all i and for all θ,

$$s_i(\theta)v(\theta_i) + t_i(\theta) \geq 0. \tag{4}$$

This means that the ruling of the system should be based on voluntary participation. No firm ever gets a negative payoff for participating in an adoption activity, i.e., for all i, $s_i(\theta)[\theta_i - v(\theta_i)] \geq 0$.

The advantage of a coordination system with monetary transfers can be shown by comparing it with a court system without monetary transfers. Formally, a court system $< s, t >$ is without transfer if for all i and for all θ, $t_i(\theta) = 0$. The following lemma shows that a monetary transfer is unavoidable in a direct court system to guarantee the combination of the absolute truth in (3) and the first-bestness.

Lemma: There is no first-best coordination system that is both absolutely truthful and without transfer.

Proof: Assume that there is such a coordination system $< s, t >$. Take a case θ and a company i such that $b_i < \theta_i < \theta_j < c_i$ for any other j's. Then firm i gets nothing from the truth-telling because of no transfer. From a false report of c_i, firm i gets $v(\theta_i) > 0$. Thus, the truth-telling does not guarantee the first-bestness. ∎

The mechanism design literature including Vickrey, Clark, and Groves, is concerned about the first-best mechanism with incentive compatibility. Especially, dominant strategy incentive compatibility has merit in that it promote the concept of absolute truthfulness in our setup. The truth-telling reports of companies result in ex post outcome efficiency. Thus, with a properly calculated transfers, the companies obtain a kind of cooperative interest by installing a pre-trial coordination system.

In this paper, we will use two important properties of the mechanism design literature. First, the net-transfer to each firm is independent of its own report. Second, the coordination system needs to play the role of brokerage in the sense that it has payments and subsidies. We propose a milder condition based on the role of brokerage in a coordination system. The monetary transfer scheme among the companies can be thought of as penalties or bonuses. If the payoffs from the coordination system with monetary transfers have the same expected payoff as in the social gain function in (1), it is acceptable as an (ex ante) bargaining arrangement by all the members. Formally, a coordination system $< s, t >$ is ex ante acceptable (EAA) if for all i,

$$E[s_i(\theta_{-i}, \theta_i)v(\theta_i) + t_i(\theta_{-i}, \theta_i)] \geq G_i \tag{5}$$

where G_i is defined in (2).

3. Main Result

For all i and for all θ, define firm i's payment at case θ as the highest valuation among the other firms,

$$p_i(\theta_{-i}) \equiv \max_{j \neq i} v(\theta_j) = v(\max_{j \neq i} \theta_j). \tag{6}$$

The last equality holds since v is a 1-1 relation. Observe that when $s_i(\theta)v(\theta_i) + t_i(\theta) = v(g(\theta)) - p_i(\theta_{-i})$ for all i and for all θ, ex post payoffs are nonnegative at any case by (1) and (6). Thus, ex post individual rationality in (4) holds. Furthermore, since $p_i(\theta_{-i})$ the gap between the social gain and individual payoff at case θ is independent of firm i's reports, there is no incentive for firm i to tell a lie. The following theorem shows how to construct a relevant transfer scheme to guarantee not only (ex post) individual rationality and the absolute truth but also ex ante acceptability; an incentive to participate into a bargaining arrangement.

Theorem: Suppose that s is an optimal assignment rule. Then there exists a monetary transfer scheme t such that $< s, t >$ satisfies the absolute truth (AT), ex post individual rationality (EPIR), and ex ante acceptability (EAA).

Proof: Set $t_i(\theta) = v(g(\theta)) - p_i(\theta_{-i}) - s_i(\theta)v(\theta_i) + K_i$ for any i and for any θ where $K_i = -E[v(g(\theta)) - p_i(\theta_{-i}) - s_i(\theta)v(\theta_i)]$. Note that $K_i = E[\mathbf{1}_{W^i}(\theta)p_i(\theta_{-i})] \geq 0$ for all i where W^i is in (2). Note also that $t_i(\theta)$ is $p_i(\theta_{-i}) + K_i$ when i is the winner, or K_i otherwise. To prove that $< s, t >$ is absolutely truthful, suppose that (3) is not hold: I.e., there exist i, θ, and θ_i' such that $s_i(\theta_{-i}, \theta_i')v(\theta_i) + t_i(\theta_{-i}, \theta_i') > s_i(\theta_{-i}, \theta_i)v(\theta_i) + t_i(\theta_{-i}, \theta_i)$. Thus, $s_i(\theta_{-i}, \theta_i')v(\theta_i) > s_i(\theta_{-i}, \theta_i)v(\theta_i)$, which is a contradiction to the optimality of s. Thus, it satisfies AT. Since, for all i, $E[s_i(\theta)v(\theta_i) + t_i(\theta)] = E[g(\theta) - p_i(\theta_{-i}) + K_i] = E[s_i(\theta)v(\theta_i)] = G_i$ in (2), $< s, t >$ is EAA. Since for all i and for all θ, $s_i(\theta)v(\theta_i) + t_i(\theta) = g(\theta) - p_i(\theta_{-i}) + K_i \geq g(\theta) - p_i(\theta_{-i}) \geq 0$, $< s, t >$ is EPIR. ∎

The transfer scheme of the above coordination system consists of two parts. The first part is a portion for being a so-called Vickrey auction [(7)] in the sense that company i's payment as a winner depends on the second highest report. The second, K_i for each i, is a portion of constant subsidy from the coordination system. The subsidy is firm-dependent.

For privatization problems with incomplete information, Dudek et al. (1995) constructed a first-best Bayesian mechanism with budget-balanced monetary transfers when agents' valuations are independent. But in many interesting cases, independent valuations are too restrictive. The result in our Theorem does not depend on the assumption of independent types. Thus, it can be widely used for any distribution F.

4. Discussion with Example: A Baby Adoption Problem

In this section, we restrict our attention to a stylized problem where there are only two pairs of client-lawyer just as in the case of the Internet Twins. A coordination system needs to decide which of two potential adopters should get the parenthood of a baby [(8)] in charge of the government. The valuation of the baby to adopters is private information

(7) See Makowski et al. (1994) and references in it.
(8) Twins for the case of the Internet Twins.

to firms. The system can assign the payment of adoption and monetary transfer between the two firms. But, the system needs to hold ex ante budget balancedness such that there would not be a monetary aid from outside.

For an indivisible prize problem with incomplete information, Dudek et al. (1995) developed a two-agent example with simple polynomial cumulative distributions of independent types. Distribution of valuations for each firm is $F_1(\theta_1) = (\theta_1)^\alpha$ and $F_2(\theta_2) = (\theta_2)^\beta$, respectively, on $[0,1] = \Theta_1 = \Theta_2$ for some $\alpha, \beta > 0$. Dudek et al. (1995) constructed a first-best Bayesian mechanism with ex post budget balancedness and ex post individual rationality for this example.

We assume that $v(\theta_i) = v \cdot \theta_i$. Then by using our Theorem, we can construct a first-best dominant-strategy mechanism with ex ante acceptability and ex post individual rationality such that the net-transfers are

$$t_1(\theta) = \begin{cases} v \cdot (-\theta_2 + \frac{\beta}{\beta+1} - \frac{\beta}{\alpha+\beta+1}), & \text{if } \theta_1 \geq \theta_2 \\ v \cdot (\frac{\beta}{\beta+1} - \frac{\beta}{\alpha+\beta+1}), & \text{otherwise,} \end{cases} \qquad (8-1)$$

$$t_2(\theta) = \begin{cases} v \cdot (\frac{\alpha}{\alpha+1} - \frac{\alpha}{\alpha+\beta+1}), & \text{if } \theta_1 \geq \theta_2 \\ v \cdot (-\theta_1 + \frac{\alpha}{\alpha+1} - \frac{\alpha}{\alpha+\beta+1}), & \text{otherwise.} \end{cases} \qquad (8-2)$$

The constant subsidy in our monetary transfer scheme depends on the parameters in the specific problem. When $\alpha = 1$ and $\beta = 2$, the constant subsidies are $\frac{1}{6}v$ for agent 1 and $\frac{1}{4}v$ for agent 2, respectively.

We formulated the case of the Internet Twins as a generalization of King Solomon's dilemma in that nobody knows the right parents of the babies. Our Groves mechanism satisfies both ex post individual rationality and ex ante acceptability by introducing sophisticated monetary transfers in a direct coordination system. From the successful solving of King Solomon's dilemma by an indirect mechanism in Perry et al. (1999), we might guess the advent of an indirect mechanism for our generalized problem. But this paper shows that at least by a direct mechanism, the problem can be successfully solved through the acceptable monetary transfers.

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