On the Arrangement of the Fixed and Contingent Attorney’s Fees

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5th Annual Meeting of Asian Law and Economics Association
In Kyun Hee University, South Korea

June 20th 2009

1 Introduction

In this paper, we model that each of risk-neutral client (a plaintiff and a defendant) offers the combination of fixed fee and contingent fee to each of risk-averse attorney. We also introduce explicitly attorney’s ability (marginal cost of attorney’s effort) in the model.

According to principal-agent theory, in general, it is mentioned that if both player are risk neutral, the principal gives agent the most incentives to eliminate agent’s moral hazard, and receives fixed amounts. Similarly, in the litigation context, attorneys (agents) buy cases from plaintiffs (principals), paying a fixed fee to the plaintiff in exchange for 100% ownership of the economic stakes in litigation. Therefore, they conduct the corner solution (except for adverse selection problem)

In the previous literatures, Baik(2008) models the situation which each of litigants (a plaintiff and a defendant) hires a attorney under the fixed and contingent fee. In his paper, all player are risk neutral and attorney’s ability is same. In his model, there are two legal system with the nonnegative fixed fee constraint and with the contingent fee cap. His conclusion is that the system with the nonnegative fixed fee gives rent to the attorneys, while the system with the contingent fee cap leave the rent to the litigant (plaintiff and defendant).

In Japan, the deregulation of the attorney’s fees started since 2004 by the Judicial Reform of Japan. According to the survey of the Judicial Reform Council of Japan, one of the reasons why the potential clients hesitate to litigate is that they are uncertain about attorney’ fees. Referring to Rules Concerning Attorney’s Fees (from 2004 in Japan),

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1See Spier(2007) and Santore and Viard(2001).
the article 2 says 'Attorney’s fees shall be appropriate and commensurate with economic benefits, complexity of the matter, time and labour required, etc.’ Therefore some clients tend to take an initiative lately in making a fee contract increasingly in Japan.

In this paper, we obtain the inner solution in regard to contingent fee (friction), which means that the contingent fee is lower and the fixed fee is higher in equilibrium if the attorney is rather risk averse, under the condition of the rather highly reservation payoff of the attorney. This could be the guideline for clients at the days of the deregulation of the attorney’s fees in Japan.

2 The Model

This paper analyzes the case that each client bilaterally retains an attorney in delegation and each attorney makes an effort for advocate client (the plaintiff) or defense client (the defendant) in contest in civil dispute, in American rule of attorney’s fees.

2.1 The Setting of Model

The players in this paper are a plaintiff, a plaintiff’s attorney (p-attorney), a defendant, and a defendant’s attorney (d-attorney). We assume each player to be risk neutral in section 2 and 3. However, we assume the plaintiff and the defendant to be risk neutral, while the p-attorney and the d-attorney to be risk averse.

After the plaintiff consults with the p-attorney, on the one hand, the plaintiff has filed a lawsuit against the defendant in order to receive compensation for her own damages. On the other hand, the defendant with the d-attorney defends against the lawsuit from the plaintiff.

The damages the plaintiff claims are denoted by $V > 0$, observable to all players. The effort level of the p-attorney is represented as $x_p (\geq 0)$, observable only to the p-attorney herself and that of the d-attorney is $x_d (\geq 0)$, observable only to the d-attorney himself. The marginal cost of her (his) effort is $c_p (\geq 0)$ for the p-attorney and $c_d (\geq 0)$ for the d-attorney respectively. The probability of winning for the plaintiff (of losing for the defendant) is defined as follows:

$$p(x_p, x_d) = \begin{cases} \frac{x_p}{x_p + x_d} & \text{for } x_p + x_d > 0 \\ \frac{1}{2} & \text{for } x_p + x_d = 0 \end{cases}$$

Note that when the each effort level is bilaterally zero, the default degree is a half. In this contest for the plaintiff’s side the plaintiff and the p-attorney can gain $V$ if she prevails, and nothing if she does not. On the contrary, for the defendant’s side the defendant and the d-attorney can defend $V$ if he prevails, and nothing if he does not.

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2This function is called logit model. See Baik and Kim (2007), etc.
In Japan, the attorney’s fees contain the fixed fee, the contingent fee or the hourly fee, etc. However, the deregulation of the attorney’s fees has started since 2004 (as one of the Judicial Reform in Japan), and then some arrangements between an attorney and a client can be the contract without the fixed fee. Presently this is not illegal in Japan.

As regard to the plaintiff side, we can describe the contingent fee that a portion of the expected judgments by court is received by the p-attorney. On the part of the defendant side, we explain it as a portion to the d-attorney of the difference between the damages claimed by the plaintiff and the expected judgments by court (the values correspond to the contribution degree of the d-attorney’s defence). The portion of the contingent fee to the p-attorney is denoted by \( \beta_p \in [0, 1] \). That to the d-attorney is by \( \beta_d \in [0, 1] \).

2.2 The Structure of the Game

Before starting the game, the plaintiff suffered the damages \( V \) from the defendant for a tort or the breach of a contract etc. The structure of the game is described as two stage as follows. In the first stage, each the plaintiff and the defendant respectively hires each attorney simultaneously. In this timing, it turns out each attorney’s ability (marginal cost of effort). We assume that switching cost of changing attorney be very high. Then, the plaintiff takes it or leaves it offer of an arrangement of an contingent fee (\( \beta_p \)) to the p-attorney and the defendant does of (\( \beta_d \)) to the d-attorney simultaneously. The offer is accepted, then the game proceeds to the next stage. In second stage, each attorney chooses an effort level simultaneously. Each effort level is observable to only each own attorney. Then, the court pass the judgment observable to all player. Each attorney accepts fees according to the attorney’s fees arrangement.

2.3 The Expected Payoff of Each Player

Each expected payoff for the plaintiff and the p-attorney is denoted by \( \Pi_p \) and \( \pi_p \), respectively. The plaintiff gains \( V \) if she prevails with the probability \( p \), nothing if she does not with \( 1 - p \), as mentioned above. As the contingent fee to the p-attorney is the portion of the expected gains, we can describe the expected payoff for the plaintiff as follows.

\[
\Pi_p = p(1 - \beta_p)V .
\] (2)

The expected payoff for the p-attorney is described as follows.

\[
\pi_p = p\beta_pV - c_px_p .
\] (3)

We can also describe each expected payoff for the defendant and the d-attorney as \( \Pi_d \) and \( \pi_d \), respectively. The defendant pays nothing if he prevails with the probability \( 1 - p \), does \( V \) if he does not with \( p \). Then the expected payments can be described as \( pV \). We

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3The author held an interview with an attorney at school of law in Kumamoto University
assume the defendant approves of paying $V$ without the d-attorney’s defence. Therefore, the contribution of the d-attorney is the differences between $V$ and $pV$. The d-attorney can receive the fraction of the contribution values. The expected payoff for defendant is as follows.

$$\Pi_d = (1 - p)(1 - \beta_d)V .$$

\hspace{1cm} (4)

The expected payoff for the d-attorney is described as follows.

$$\pi_d = (1 - p)\beta_d V - c_d x_d .$$

\hspace{1cm} (5)

Following the setting above, we use the backward induction in order to solve the game.

2.4 The Second Stage Solution

In the second stage, each attorney expends some efforts simultaneously, given that each contract of attorney’s fees ($\beta_p$, $\beta_d$) is determined. Anticipating the level of the d-attorney’s effort, the maximum problem of the p-attorney’s expected payoff is as follows.

$$\max_{x_p} \pi_p = \left( \frac{x_p}{x_p + x_d} \right) \beta_p V - c_p x_p$$

\hspace{1cm} (6)

We obtain the reaction function of the p-attorney by the first order condition as follows.

$$x_p = - x_d + \sqrt{ \frac{\beta_p}{c_p} V x_d }$$

\hspace{1cm} (7)

This reaction function contains the strategic complemental part and the strategic substitutive part. Note that this reaction function is not included origin. Because the p-attorney exerts effort slightly if the d-attorney’s effort level is zero, so that the the plaintiff can be prevailing, on the basis of the definition of the winning probability for the plaintiff.

Next we consider the maximum problem of the d-attorney’s expected payoff, anticipating the level of the p-attorney’s effort as follows.

$$\max_{x_d} \pi_d = \left( \frac{x_d}{x_p + x_d} \right) \beta_d V - c_d x_d$$

\hspace{1cm} (8)

the reaction function of the d-attorney is obtained by first order condition as follows.

$$x_d = - x_p + \sqrt{ \frac{\beta_d}{c_d} V x_d }$$

\hspace{1cm} (9)

Now we can obtain the Nash equilibrium for each effort level of the p-attorney and the d-attorney in the subgame. From the equation (7) and the equation (9), each equilibrium effort level in the second stage of the game is as follows.

$$\{ x^*_p , x^*_d \} = \left\{ \frac{\beta_p^2 \beta_d c_d V}{(\beta_p c_d + \beta_d c_p)^2} , \frac{\beta_p \beta_d^2 c_p V}{(\beta_p c_d + \beta_d c_p)^2} \right\}$$

\hspace{1cm} (10)

We can illustrate each reaction function and some equilibrium points due to some parameters in Figure-1. Here we can refer to the specific characteristics of the reaction function as lemma 1.
Lemma 1. The higher effort for one attorney, the other attorney make a higher effort up to some level, on the contrary, a lower effort in excess of it. Because the increase of the opponent’s effort drives the other to make higher effort, but to do lower effort where marginal prevailing of effort is less than marginal cost of it.

Now differentiating the expression (10) yields the following comparative statics with regard to each equilibrium effort level obviously.

\[
\frac{\partial x_p^*}{\partial V} > 0 \ , \ \frac{\partial x_d^*}{\partial V} > 0 \ , \ \frac{\partial x_p^*}{\partial \beta_p} > 0 \ , \ \frac{\partial x_d^*}{\partial \beta_d} > 0
\]  

(11)

Let us denote the expression (11) as Lemma.

Lemma 2. The greater damages means that each attorney makes a higher effort. And the higher fraction of contingent fee for each attorney, the higher effort level is achieved by him.

In contrast, the cross effect of each attorney’s fees to each equilibrium effort level (the sign of \(\frac{\partial x_p^*}{\partial \beta_d}\) and \(\frac{\partial x_d^*}{\partial \beta_p}\)) is dependent on some parameters. We examines this effect as follows.

The case [I]: \(\beta_p/c_p \geq \beta_d/c_d\)
In this case, that is, the portion of the p-attorney’s fee to her marginal cost is greater than
the d-attorney’s, we can obtain the equilibrium composed of the strategic complemental
part of the p-attorney’s reaction function and substitutive one of the d-attorney’s. The
equilibrium is satisfied by the following conditions.

\[ x_p^* \geq \frac{\beta_d V}{4c_d}, \quad x_d^* \leq \frac{\beta_p V}{4c_p} \]  

(12)

Substituting the expressions (10) for the conditions (12), we can obtain the following.

\[ \beta_d \leq \frac{c_d}{c_p} \beta_p \]  

(13)

We can show the partition of this region. Then we make sure of the sign of \( \frac{\partial x_p^*}{\partial \beta_d} \) and
\( \frac{\partial x_d^*}{\partial \beta_p} \). Firstly, differentiating \( x_p^* \) of the expression (10), we can obtain as follows.

\[ \frac{\partial x_p^*}{\partial \beta_d} = \frac{(\beta_p c_d - \beta_d c_p) \beta_d c_d V}{(\beta_p c_d + \beta_d c_p)^3} \]  

(14)

As we use the condition (13), we can obtain the following.

\[ \frac{\partial x_p^*}{\partial \beta_d} \geq 0, \quad if \quad \frac{\beta_p}{c_p} \geq \frac{\beta_d}{c_d} \]  

(15)

Secondly, in a similar fashion as the above case, we can obtain as follows.

\[ \frac{\partial x_d^*}{\partial \beta_p} \leq 0, \quad if \quad \frac{\beta_p}{c_p} \geq \frac{\beta_d}{c_d} \]  

(16)

Now we describe this implication as lemma as follows.

**Lemma 3.** Suppose the rate of the p-attorney’s fee in respect to the marginal
cost of her own effort is greater than the one of the d-attorney’s, that is, the p-
attorney talentedly superior to the d-attorney. The defendant side raising the
d-attorney’s fee, the p-attorney increases her own effort level, on the contrary,
the higher for the p-attorney’fee, then the d-attorney decrees his own effort
level.

Next we examine the other case as follows.

**The case [II] : \( \beta_p/c_p \leq \beta_d/c_d \)**

In this case that is, the portion of the d-attorney’s fee to his marginal cost is greater than
the p-attorney’s, we can obtain the equilibrium composed of the strategic substitutive
part of the p-attorney’s reaction function and complementary one of the d-attorney’s.
The equilibrium is satisfied by the following conditions.

\[ x_p^* \leq \frac{\beta_d V}{4c_d}, \quad x_d^* \geq \frac{\beta_p V}{4c_p} \]  

(17)
In a similar fashion as case [1], we can obtain as follows.

\[ \beta_d \geq \frac{c_d}{c_p} \beta_p \]  

(18)

In this part, the sign of \( \partial x^*_d / \partial \beta_p \) and \( \partial x^*_p / \partial \beta_d \) is as follows.

\[ \frac{\partial x^*_d}{\partial \beta_p} \geq 0 , \quad \text{if } \frac{\beta_p}{c_p} \leq \frac{\beta_d}{c_d} \]  

(19)

\[ \frac{\partial x^*_p}{\partial \beta_d} \leq 0 , \quad \text{if } \frac{\beta_p}{c_p} \leq \frac{\beta_d}{c_d} \]  

(20)

Therefore, we can describe this meaning as lemma as follows.

**Lemma 4.** Suppose the rate of the d-attorney’s fee to the marginal cost of his own effort is higher than the one of the p-attorney’s, that is, the d-attorney talentedly superior to the p-attorney. Then as the plaintiff’s side raising the p-attorney’s fee, the d-attorney increases his own effort level, on the contrary, the higher for the d-attorney’s fee, then the p-attorney decreses her own effort level.

### 3 Determination of the Attorney’s Fees

We read in the equilibrium effort level of each attorney in the second stage. Then each of the plaintiff and the defendant should solve the expected payoff maximizing problem simultaneously in the first stage. we examine the winning probability for the plaintiff, and the expected payoff for each attorney in the equilibrium of the second stage.

#### 3.1 The Winning Probability In Equilibrium

Beforehand, we represent the comparison of the p-attorney’s marginal cost and the d-attorney’s by ratio in the following form.

\[ \frac{c_p}{c_d} = h, \quad \text{or } \frac{c_d}{c_p} = k \]  

(21)

This means the relative ability of each attorney.

Now, substituting the solution (10) in the second stage for the expression(1), the probability of winning for the plaintiff in the equilibrium can be obtained as follows.

\[ p^* = \frac{\beta_p}{\beta_p + h \beta_d} \]  

(22)

Therefore, we can obtain the following easily.

\[ \frac{\partial p^*}{\partial \beta_d} \leq 0 , \quad \frac{\partial p^*}{\partial \beta_p} \geq 0 , \quad \frac{\partial p^*}{\partial h} \leq 0 \]  

(23)
These mean that in the equilibrium the higher the d-attorney’s fee, the winning probability for the plaintiff falls down, on the contrary the higher the p-attorney’s one, the probability grows up, and the lower p-attorney’s ability, the winning probability for the plaintiff goes down.

### 3.2 The Expected Payoff for Each Attorney in Equilibrium

Now let us examine the expected payoff for each attorney in equilibrium. Firstly, substituting the expression (10) for the payoff for the p-attorney (3), we can obtain the following.

\[ \pi^*_p = \frac{\beta_p^3 V}{(\beta_p + h \beta_d)^2} \]  

(24)

Therefore, we can take the comparative statics as follows.

\[ \frac{\partial \pi^*_p}{\partial \beta_p} > 0, \quad \frac{\partial \pi^*_p}{\partial \beta_d} < 0, \quad \frac{\partial \pi^*_p}{\partial V} > 0, \quad \frac{\partial \pi^*_p}{\partial h} < 0 \]  

(25)

Secondly, in similar fashion as above, we can obtain the following expected payoff for the d-attorney in equilibrium.

\[ \pi^*_d = \frac{\beta_d^3 V}{(k \beta_p + \beta_d)^2} \]  

(26)

The comparative statics is then derived as follows.

\[ \frac{\partial \pi^*_d}{\partial \beta_d} > 0, \quad \frac{\partial \pi^*_d}{\partial \beta_p} < 0, \quad \frac{\partial \pi^*_d}{\partial V} > 0, \quad \frac{\partial \pi^*_d}{\partial k} < 0 \]  

(27)

We summarize this interpretation as lemma as follows.

**Lemma 5.** In equilibrium, the greater the damages, the expected payoff for each attorney increase. The higher the opponent attorney’s fees, the expected payoff for each attorney decrease, and vice versa. And also in equilibrium, the higher his (her) own attorney’s ability, the payoff of each attorney increase.

### 3.3 The First Stage Solution

Now we consider the contracts of the plaintiff and the defendant satisfied with each attorney’s participating constraint. Firstly, we examine the contract of the plaintiff side. Substituting the expression (10) for the plaintiff’s expected payoff (2), and here should the reservation payoff be zero for simplicity, we can obtain the maximaizing problem for the plaintiff by using the equation (22) as follows.

\[ \max_{\beta_p} \Pi_p = \left( \frac{\beta_p}{\beta_p + h \beta_d} \right) (1 - \beta_p) V, \]  

(28)

\[ s.t. \quad \pi^*_p = \frac{\beta_p^3 V}{(\beta_p + h \beta_d)^2} \geq 0 \]  

(29)
The constraint condition (29) is obviously satisfied. Therefore, we can calculate the first order condition of the equation (28) throwing the condition (29) away. The following is derived from the first order condition
\[ \beta_p^2 + 2\beta_p \beta_d - h \beta_d = 0. \tag{30} \]

Therefore, we can obtain the plaintiff’s reaction function as follows.
\[ \beta_p = - h \beta_d + \sqrt{h^2 \beta_d^2 + h \beta_d} \tag{31} \]

Let us examine the shape of the plaintiff’s reaction function. Then differentiating \( \beta_p \) with \( \beta_d \) of the equation (31), we can obtain the following form without difficulty.
\[ \frac{\partial \beta_p}{\partial \beta_d} > 0, \quad \frac{\partial^2 \beta_p}{\partial \beta_d^2} < 0 \tag{32} \]

We can also examine the effect of the parameter \( h \) to the plaintiff’s reaction function.
\[ \frac{\partial \beta_p}{\partial h} = \frac{-2\beta_p \sqrt{h^2 \beta_d^2 + h \beta_d} + 2\beta_d^2 + \beta_d}{2h \sqrt{h^2 \beta_d^2 + h \beta_d}} > 0 \tag{33} \]

The positive sign of the numerator can be confirmed without difficulty. These graphs of the plaintiff’s reaction function are illustrated in Figure 2.

Secondly, we focus on the fee contract of the defendant side. We can formalize the maximizing problem for the defendant as follows.
\[
\max_{\beta_d} \Pi_d = \left( \frac{\beta_d}{k \beta_p + \beta_d} \right) (1 - \beta_d) V, \tag{34}
\]
\[ \text{s.t. } \pi_d^* = \frac{\beta_d^2 V}{(k \beta_p + \beta_d)^2} \geq 0 \tag{35} \]

In the similar fashion as the plaintiff side, we can obtain the defendant’s reaction function in the following form.
\[ \beta_d = - k \beta_p + \sqrt{k^2 \beta_p^2 + k \beta_p} \tag{36} \]

We examine the shape of the defendant’s reaction function in the similar method as above. The following form can be obtained.
\[ \frac{\partial \beta_d}{\partial \beta_p} > 0, \quad \frac{\partial^2 \beta_d}{\partial \beta_p^2} < 0 \tag{37} \]

The effect of the parameter \( k \) to the defendant’s reaction function is derived as follows in the similar fashion as the plaintiff side.
\[ \frac{\partial \beta_d}{\partial k} = \frac{-2\beta_p \sqrt{k^2 \beta_p^2 + k \beta_p} + 2\beta_p^2 + \beta_p}{2 \sqrt{k^2 \beta_p^2 + h \beta_p}} > 0 \tag{38} \]
Figure-2: The Locus of the Equilibrium

We illustrate these graph of the defendant’s reaction function in Figure 2.

Now we should solve the Nash equilibrium for each attorney’s fees in total game. We derive the intersection from both the plaintiff and the defendant reaction function. We can obtain the locus of intersection of both the reaction functions, solving the equation (31) and (36) with respect to $h$ as follows.

$$\beta_d = \frac{2\beta_p - 1}{2\beta_p - 2}$$  (39)

This locus is illustrated in Figure 2. Let us specify the parameters now. If $h = k = 1$, that is, $c_p = c_d$, this case means that the p-attorney’s ability is equal to the d-attorney’s. Then in this case we represent $((\beta_p^*, \beta_d^*) = (1/3, 1/3))$ as equilibrium. The locus in north-west implies that the p-attorney is superior to the d-attorney in ability. The one in south-east means vice-versa. Therfore, we can understand the following properties by the the parameters comparatives.

$$\frac{\partial \beta_p^*}{\partial h} > 0, \quad \frac{\partial \beta_d^*}{\partial h} < 0, \quad \frac{\partial \beta_d^*}{\partial k} > 0, \quad \frac{\partial \beta_p^*}{\partial k} < 0$$  (40)

We summarize these results as the proposition as follows.

**Proposition 1**

In equilibrium, the more superior the opponent attorney to his own, the client

\[\text{The second order condition easily can be confirmed.}\]
makes a fee contract with his attorney to raise his own attorney’s fees up. On the contrary, the more inferior the opponent, the client does a fee contract to decrease his own attorney’s fees.

4 Fixed and Contingent Fee with Risk Attitude

In this section, we introduce the fixed fee ($t_p$ for plaintiff’s side and $t_d$ for defendant’s side), and the degree of the attorneys’ attitude with risk averse. Then we use the parameter $\theta_p \in (0, 1]$ for p-attorney and $\theta_d \in (0, 1]$ for d-attorney as the degree of it. These parameter being close to zero means that the attorney is rather risk averse relatively. On the contrary, we assume the plaintiff and the defendant are risk neutral. Then, we solve this problem by backward induction.

4.1 Risk-Averse Attorney Contest

We formulate the expected payoff of p-attorney with risk-averse ($\pi^r_p$) as follows.

$$\pi^r_p = \left(\frac{x_p}{x_p + x_d}\right)(\beta_p V)^{\theta_p} - c_p x_p + t_p$$

(41)

We can obtain the first order condition in the following equation.

$$x_p = -x_d + \sqrt{\frac{x_d (\beta_p V)^{\theta_p}}{c_p}}$$

(42)

Next, the the expected payoff of d-attorney with risk-averse ($\pi^r_d$) is done as follows.

$$\pi^r_d = \left(\frac{x_d}{x_p + x_d}\right)(\beta_d V)^{\theta_d} - c_d x_d + t_d$$

(43)

The first order condition is obtained in the following equation.

$$x_d = -x_p + \sqrt{\frac{x_p (\beta_d V)^{\theta_d}}{c_d}}$$

(44)

These reaction functions are almost same shape as previous section.

4.2 The Second Stage Solution

We solve the Nash equilibrium in the second stage simultaneously. Then we can obtain the equilibrium effort of each attorney $\{x^r_p, x^r_d\}$.

$$\{x^r_p, x^r_d\} = \left\{ \frac{c_d (\beta_p V)^{2\theta_p} (\beta_d V)^{\theta_d}}{(c_d (\beta_p V)^{\theta_p} + c_p (\beta_d V)^{\theta_d})^2}, \frac{c_p (\beta_p V)^{\theta_p} (\beta_d V)^{2\theta_d}}{(c_d (\beta_p V)^{\theta_p} + c_p (\beta_d V)^{\theta_d})^2} \right\}$$

(45)
4.3 The Probability and the Expected Payoff of Each Attorney

We can figure the plaintiff’s winning probability \( p_r \), the p-attorney’s expected payoff \( \pi_p \), and the d-attorney’s expected payoff \( \pi_d \) by reading in the second stage solution.

\[
p_r = \frac{x_p}{x_p + x_d} = \frac{(\beta_p V)^{\theta_p}}{(\beta_p V)^{\theta_p} + h(\beta_d V)^{\theta_d}} \quad (46)
\]

\[
\pi_p = p_r (\beta_p V)^{\theta_p} - c_p x_p + t_p = \frac{(\beta_p V)^{3\theta_p}}{(\beta_p V)^{\theta_p} + h(\beta_d V)^{\theta_d})^2} + t_p \quad (47)
\]

\[
\pi_d = (1 - p_r)(\beta_d V)^{\theta_d} - c_d x_d + t_d = \frac{(\beta_d V)^{3\theta_d}}{(\beta_p V)^{\theta_p} + (\beta_d V)^{\theta_d})^2} + t_d \quad (48)
\]

4.4 The Plaintiff’s Payoff Maximaization Problem

Therefore, we can formulate the plaintiff payoff maximaization problem as follows, where \( \hat{\pi}_p \) (constant) is reservation payoff for the p-attorney.

\[
\max_{\beta_p} \Pi_p = \left( \frac{(\beta_p V)^{\theta_p}}{(\beta_p V)^{\theta_p} + h(\beta_d V)^{\theta_d})^2} \right) (1 - \beta_p) V - t_p \quad (49)
\]

s.t. \( \pi_p = \frac{(\beta_p V)^{3\theta_p}}{(\beta_p V)^{\theta_p} + h(\beta_d V)^{\theta_d})^2} + t_p \geq \hat{\pi}_p \quad (50)
\]

Then, we bind the constraint condition to equality, we obtain the fixed fee as follows.

\[
t_p = \pi_p - \frac{(\beta_p V)^{3\theta_p}}{(\beta_p V)^{\theta_p} + h(\beta_d V)^{\theta_d})^2} \quad (51)
\]

We substitute the equation (51) into the plaintiff’s payoff function (49). Therefore the plaintiff’s payoff maximazation problem is as follows.

\[
\max_{\beta_p} \Pi_p = \left( \frac{(\beta_p V)^{\theta_p}}{(\beta_p V)^{\theta_p} + h(\beta_d V)^{\theta_d})^2} \right) (1 - \beta_p) V - \hat{\pi}_p + \frac{(\beta_p V)^{3\theta_p}}{(\beta_p V)^{\theta_p} + h(\beta_d V)^{\theta_d})^2} \quad (52)
\]

Therefore, we can obtain the first order condition for the plaintiff as following form.

\[
- \beta_p V \{(\beta_p V)^{\theta_p} + h(\beta_d V)^{\theta_d})^2 + \theta_p(1 - \beta_p)h^2 V(\beta_d V)^{2\theta_d} \]
\[+ (\beta_p V)^{3\theta_p} + (\beta_p V)^{\theta_p}h(\beta_d V)^{\theta_d}(V - \beta_p V + 3(\beta_p V)^{\theta_p}) = 0 \quad (53)
\]

4.5 The Defendant’s Payoff Maximaization Problem

We can obtain the defendant’s payoff maximazation problem as same way of the plaintiff, where \( \hat{\pi}_d \) (constant) is reservation payoff for the d-attorney.
\[
\max_{\beta_d} \Pi_d = \left(1 - \frac{(\beta_p V)^{\theta_p}}{(\beta_p V)^{\theta_p} + h(\beta_d V)^{\theta_d}}\right) (1 - \beta_d) V - t_d \\
\text{s.t.} \quad \pi^*_d = \frac{(\beta_d V)^{3\theta_d}}{\left(\frac{1}{h}(\beta_p V)^{\theta_p} + (\beta_d V)^{\theta_d}\right)^2} + t_d \geq \hat{\pi}_d^*
\]

As same as the plaintiff case, we bind the constraint condition to equality, we obtain the fixed fee for the defendant in the following form.

\[
\max_{\beta_d} \Pi_d = \frac{(\beta_d V)^{\theta_d}}{\left(\frac{1}{h}(\beta_p V)^{\theta_p} + (\beta_d V)^{\theta_d}\right)} (1 - \beta_d) V - t_d + \frac{(\beta_d V)^{3\theta_d}}{\left(\frac{1}{h}(\beta_p V)^{\theta_p} + (\beta_d V)^{\theta_d}\right)^2} \geq 0
\]

We substitute the equation (56) into the defendant’s payoff function (54). Therefore the defendant’s payoff maximization problem is as follows.

\[
\text{We take an example of two case about the attorney’s risk attitude, } \theta = \{\theta, \bar{\theta}\}, (\theta < \bar{\theta}). \text{ Specifically let us done, } \theta = 0.5, \bar{\theta} = 0.75. \text{ As taking the numerical examples, then we can obtain equilibrium points } (\beta^*_p, \beta^*_d) \approx (0.2057, 0.2057) \text{ if } \theta = 0.5, \text{ and } (\beta^*_p, \beta^*_d) \approx (0.3133, 0.3133) \text{ if } \bar{\theta} = 0.75. \text{ Therefore, we understand the inner solution can be obtain if the attorney is risk averse. In Figure 3, there are those reaction functions of the plaintiff and the defendant and two intersections of both symmetric reaction functions. The reaction functions of both the plaintiff and the defendant are strategic complement, which means that the greater the opponent’s contingent fee, the contingent fee in the own side should be increased in the best response.}
\]

4.6 Determination of Fixed and Contingent Fee in the First Stage

Now we must derive the equilibra in the first stage, however it would be difficult to seek them analytically in general unless some paremeters is specified. Let us in advance specific some parameter, \(h = 1, \theta_p = \theta_d = \theta, \ (\theta \in (0, 1]) \) which means each attorney’ ability being equal and each attorney’s risk attitude being same. We compare rather risk averse with a little risk averse.

**Numerical Analysis with Risk Averse Case**

We take an example of two case about the attorney’s risk attitude, \(\theta = \{\theta, \bar{\theta}\}, (\theta < \bar{\theta}). \) Specifically let us done, \(\theta = 0.5, \bar{\theta} = 0.75. \) As taking the numerical examples, then we can obtain equilibrium points \((\beta^*_p, \beta^*_d) \approx (0.2057, 0.2057) \) if \(\theta = 0.5, \) and \((\beta^*_p, \beta^*_d) \approx (0.3133, 0.3133) \) if \(\bar{\theta} = 0.75. \) Therefore, we understand the inner solution can be obtain if the attorney is risk averse. In Figure 3, there are those reaction functions of the plaintiff and the defendant and two intersections of both symmetric reaction functions. The reaction functions of both the plaintiff and the defendant are strategic complement, which means that the greater the opponent’s contingent fee, the contingent fee in the own side should be increased in the best response.
Comparatively, the south-west symmetric point is the one when both attorney is rather risk averse. Therefore, we understand that as \( \theta \) decrease, both \( (\beta_p^r, \beta_d^r) \) goes down, that is, the more risk averse of both attorney, equilibrium contingent fee decrease.

We must reference the fixed fee. Assuming the two case of \( \theta \) above, we can compare the fixed fee about the attorney’s risk attitude. In the symmetric setting above, we shall rewrite the equation of the fixed fee, (51) or (56) as follows.

\[
t(\theta) = \hat{\pi} - \frac{1}{4} (\beta^r V)^\theta \quad \text{if} \quad \theta = \theta, \quad t(\bar{\theta}) = \hat{\pi} - \frac{1}{4} (\bar{\beta}^r V)^\bar{\theta} \quad \text{if} \quad \theta = \bar{\theta}.
\]

As \( (\beta V)^\theta < (\bar{\beta} V)^\bar{\theta} \) holds if and only if \( \beta V > 1 \), then we understand that \( t(\theta) \) is gerater than \( t(\bar{\theta}) \) if \( t(\cdot) \) is positive. Therefore, the equilibrium contingent fee is lower and the equilibrium fixed fee is higher when the attoney is rather risk averse, in the system of the nonnegative fixed fee. We summarize these results as proposition.

**Proposition 2**

In equilibrium, when the attoney is rather risk averse, the contingent fee is lower and the fixed fee is higher under the condition of the rather highly reservation payoff of the attorney.

5 Concluding Remark

In this paper, we modeled that each of the risk-neutral plaintiff and defendant offers the combination of fixed fee and contingent fee to each of risk-averse attorney. We also
introduce explicitly attorney’s ability in the model. What we should think is the access of clients to attorneys under the recent judicial reform of Japan and deregulation of attorney’s fees. As the further research, the assignments are hourly fee, conditional fee, legal aid and so on.
References


