The Economics of the Fourth Amendment: 
Crime, Search, and Anti-Utopia

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Abstract

We develop a strategic model of crime and punishment. We then give the Fourth Amendment formal expression in terms of the model, and analyze its effect on equilibrium elements of social welfare. In one parameter range, the Fourth Amendment actually increases police search, and has an ambiguous effect on wrongful searches. But in the other intermediate range, it reduces police search and wrongful searches. In both ranges, it increases crime but reduces wrongful convictions. Moreover, a strong Fourth Amendment and strict police accountability are jointly sufficient for ongoing progress in search technology to ultimately lead to an extreme parameter range where the stable equilibrium is Utopian, in the sense that the police never search without probable cause and most citizens do not commit crime.

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BIG BROTHER IS WATCHING YOU

–George Orwell, 1984, p.4

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1. Introduction

The Fourth Amendment to the United States Constitution guarantees that “[The] right of the people to be secure in their persons, houses, papers, and effects, against unreasonable searches and seizures, shall not be violated, and no Warrants shall issue, but upon probable cause, supported by Oath or affirmation, and particularly describing the place to be searched, and the persons or things to be seized.” The Fourth Amendment protects citizens against the invasion of their privacy by agencies of the U.S. government. In particular, suspects in a crime have a constitutional right not to be searched by the police unless the evidence against them constitutes probable cause.

This does not mean the police never search without probable cause. In practice, most criminal court cases start with preliminary hearings with motions by defense to suppress evidence on the grounds that the search that uncovered it was warrantless, or was conducted without probable cause. The police often have to make a determination of probable cause in a haste because evidence is most incriminating right after the crime has been committed. Thus they cannot be directly prevented from searching without probable cause, but if they are subsequently found to have done so, any evidence that they uncovered in this way may be excluded at trial. This is the Fourth Amendment’s exclusionary rule.

2 See, for example, Gunther and Sullivan [1997, Appendix A].
3 In practice, probable cause exists when it is more likely than not (more than 50 percent certainty) that the items to be seized are connected to the crime and that they can be found in the places to be searched. The legal definition of probable cause was formulated in Brinegar v. US [338 U.S. 839, 1949]:

Probable cause, such as may justify an arrest or a search and seizure without warrant, is a reasonable ground for belief of guilt; and this means less than evidence which would justify condemnation or conviction; probable cause exists where the facts and circumstances within the knowledge of the officer making the arrest or search, and of which he had reasonably trustworthy information, are sufficient in themselves to warrant a man of reasonable caution in the belief that an offense has been or is being committed.

4 Precedent for the Fourth Amendment’s exclusionary rule was set in Mapp v. Ohio [367 U.S. 643, 1961]. The exclusionary rule is also part of the criminal justice systems of many other industrialized countries. For example, Section 8 of the Canadian Charter of Rights and Freedoms declares that “Everybody has the
In its war against drugs, terrorism, and crime in general, the U.S. government has gradually weakened Fourth Amendment protections of this kind against invasions of privacy by the police. One example of new legislation in this direction is the USA Patriot Act, which was passed by Congress shortly after the September 11, 2001, terrorist attacks. A provision in the Act gives the Federal Bureau of Investigation authority to search library and bookstore circulation records and, where applicable, Internet user records, in an investigation of international terrorism [see McCarthy, 2002, for an analysis of the Patriot Act].

This paper seeks to understand the effects of the Fourth Amendment’s exclusionary rule on social welfare as a first step toward determining the optimal extent to which Fourth Amendment rights should be sacrificed for increased security against crime and terrorism. A game-theoretic model is developed to understand the welfare effects of the Fourth Amendment. In the model, crime is endogenous, and the police choose whether or not to search suspects when the evidence against them does not constitute probable cause. The model is solved for its equilibrium crime, search, and conviction rates, with stronger and weaker Fourth Amendment protections, which are assumed to reduce the ultimate probability of conviction in cases where the police searched a suspect without probable cause.

Several interesting results emerge. In accordance with intuition, the Fourth Amendment increases crime but reduces wrongful convictions. However, it has two conflicting effects on the police’s equilibrium search intensity without probable cause. First, it tends to decrease police search without probable cause directly by reducing the probability that such searches lead to successful conviction. Second, it tends to increase police search indirectly by increasing crime. If the direct effect dominates, an increase in Fourth Amendment protections

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right to be secure against unreasonable searches and seizures” (see, for example, Yegis [2003]). And Section 24(1) of the Charter provides that anyone who has rights that have been infringed may apply to a court to obtain an appropriate remedy. The remedy that the court can apply is to exclude the evidence obtained in the course of a violation of a constitutional right. This has become known as the “Canadian Constitutional exclusionary rule.”

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reduces police search and reduces wrongful searches. But if the indirect effect dominates, an increase in Fourth Amendment protections increases police search, and has an ambiguous effect on wrongful searches. The Fourth Amendment is more likely to reduce wrongful searches if the police are accountable to the people for their mistakes, in the sense that they suffer a loss whenever they invade the privacy of the innocent. Thus the Fourth Amendment is more likely to reduce wrongful searches if government officials are democratically elected, citizens have liberties of speech and assembly, and the press is free, each of which tend to make government officials and the police more accountable to the people. Moreover, unlike the Fourth Amendment, police accountability unambiguously reduces wrongful searches.

To derive the above results, we focus on the only equilibrium in the model that is both stable, in the sense that it survives small perturbations in the equilibrium strategies, and non extreme, in the sense that the police do not search with probability 0 or 1 when the evidence is not probable cause. However, the model also has two extreme equilibria that are nonetheless stable, one in which the police always search without probable cause and most citizens commit crime, what we call an anti-Utopian equilibrium, and one in which the police never search without probable cause and most citizens do not commit crime, what we call a Utopian equilibrium. We argue that ongoing progress in search technology will ultimately lead the world from the stable, non-extreme equilibrium to either the Utopian or anti-Utopian equilibrium. We then prove that progress in search technology leads to the anti-Utopian equilibrium if the police are not accountable to the people; but that it leads to the Utopian equilibrium if the police are accountable to the people and strong Fourth Amendment protections are in place.

The next section relates the contribution to the existing economics literature. Section 3 presents the model, and formally defines Fourth Amendment protections. Section 4 analyzes
the effect of the Fourth Amendment on equilibrium and elements of welfare. Section 5 compares the predictions under differing degrees of police accountability. Section 6 analyzes the implications of long term progress in search technology. Section 7 summarizes and suggests avenues for further research.

2. Related Literature

The economics literature on crime and policing is vast, starting with Becker [1968] and Ehrlick [1973]. For a survey, see Ehrlich [1996]. However, this literature has rarely considered the strategic interaction between criminals and the police. One exception is Persico [2002], who develops a model of racial profiling that takes into account the behavior of both criminals and the police. In his model, the objective of the police is to maximize the number of successful searches. Police observe citizens’ race, but not their earning opportunities in the legal sector, before deciding whether or not to search them. In equilibrium, the group with lower legal opportunities is searched with greater intensity than the group with higher opportunities. Persico then finds conditions under which forcing the police to equalize its search intensities across the two groups reduces crime.

Persico’s focus is on fairness issues that concern the Fourteenth Amendment, which aims to protect against racial discrimination, not the Fourth Amendment, which aims to protect against wrongful searches. Rubin and Atkins [1999] is the only existing manuscript that analyzes the Fourth Amendment using economic theory. The authors sketch a Beckerian model in which the Fourth Amendment reduces search and increases crime. In their model, the police make their decisions to search suspects without worrying about how their decisions will affect citizens’ decisions to commit crime. In our model, citizens’ decisions to commit crime influence the police’s decisions to search, and vice versa. We solve for the mixed-
strategy Bayesian equilibria of this interaction. Developing an explicitly strategic model of crime and punishment is one of the main contributions of our paper.

Our model is also designed to explore the effects of the Fourth Amendment when democracy and First Amendment rights make the police accountable for their mistakes, and in the distant future, when search technologies will have become so advanced that the police could employ them to search citizens at every moment. Thus our contribution is also connected, although only in spirit, to the literatures on the economics of government accountability [see Mashaw, 1978, Seabright, 1996, and Owen, 1974], and the economics of Utopia and anti-Utopia [see Jonas, 1981, Roback, 1985, Hodgson, 1995, Foss, 2000].

3. Strategic Model of Crime and Punishment

The model’s actors are a unit mass of citizens and a single, coordinated police force. Citizens differ according to their benefit or wage from crime, $w_C$. At time 1, Nature chooses each citizen’s $w_C$ according to a cumulative density function $F(w_C)$, which is assumed to be the uniform distribution defined on the $[0, 1]$ interval.\footnote{The choice of a uniform distribution is entirely for the sake of computational simplicity. The qualitative results presented in the paper are valid for a general distribution. We have shown this in a technical appendix available on request.} This density function is common knowledge, but the police do not learn its realization.

At time 2, citizens choose an action from the set $\{C, \neg C\}$, that is, they each choose whether or not to commit a crime ($C$). Their choices at time 2 are not observable to the police. At time 3, Nature chooses the preliminary evidence $\varepsilon$. The random variable $\varepsilon$ can be in one of two states, $I_\varepsilon$, meaning that the evidence against the citizen is not probable cause, or $G_\varepsilon$, meaning that this evidence is probable cause.

The quality of the evidence is represented by the parameter $P[I_\varepsilon \neg C]$, the probability that the evidence is not probable cause given that the citizen did not commit the crime,
and the parameter $P[G_\varepsilon|C]$, the probability that the evidence is probable cause given that the citizen did commit the crime. Let $P[I_\varepsilon|\neg C] = P[G_\varepsilon|C] = q$, that is, assume that guilty citizens are as likely to generate evidence that is probable cause as innocent citizens are to generate evidence that is not probable cause. Assume $q > \frac{1}{2}$, that is, the evidence is more often right than wrong for all crimes.

At time 4, Nature chooses the police’s knowledge of the evidence. This random variable can be in one of two states, either the evidence against a citizen comes to the police’s attention or it does not. The police are no more likely to come across the evidence if it constitutes probable cause than if it does not. Let $\pi$ denote the unconditional probability that the evidence comes to the police’s attention, which may be larger as the size of the police force grows relative to the total population or the crimes considered are higher profile.

If the evidence does not come to the police’s attention, the game is over. If it comes to their attention, then at time 5, the police choose an action from the set $\{S, \neg S\}$, that is, they choose whether or not to search ($S$) the citizen’s property. When the police make their decision, they only know whether or not the evidence is probable cause, that is, whether $I_\varepsilon$ or $G_\varepsilon$. If the police have probable cause, they always choose to search. More precisely, if the police learn $G_\varepsilon$, they always choose $S$.

The police incur a cost $c^S$ to search a citizen’s property. Innocent citizens incur a cost $\eta_I$ of being searched, which measures the extent to which they value their privacy. But why do citizens value their privacy and why are they harmed when it is invaded by the police?

Privacy may be interpreted as the ability to conceal personal information that others might use to one’s disadvantage. Concealment serves the important purpose of protecting reputation, which is often a valuable asset in relationships. If citizens are searched by the police and the details of the search are subsequently made public, they may suffer a loss of
reputation, which might result in them losing their job or their spouse. This is the meaning of privacy underlying the federal Privacy Act [5 U.S.C. Chapter 552 (a), 1976], which limits the retention and dissemination of discrediting personal information in government files.\textsuperscript{6}

But invasion of privacy is also damaging because it breaks a social bond of trust. If parents invade their children’s privacy, say by searching their rooms, to ensure that they do not smoke, then they are signaling to their children that they do not trust them. And the children might rebel against their parents and start smoking just to spite them or to assert their independence, even if they are the type of children who would not have smoked in the absence of excessive monitoring.

Similarly, if the police install surveillance cameras in the house of every citizen and provide strong incentives for citizens to tattle on each other, to ensure that none of them ever commit a crime, then citizens may feel that many other aspects of their life are also constrained, and perhaps worse, they may feel that they have lost the trust of their neighbors and the government. In these circumstances, they may choose to rebel and commit more crime. In the next section, we will see that rebellion of this kind actually occurs in several important, albeit extreme, regions of the model’s parameter space.

For the innocent citizens, the costs of having their privacy invaded, loss of reputation, freedom, and others’ trust, are exacerbated by feelings that these penalties were not deserved. It is not simply that they have to pay a penalty, but that they have to pay a penalty for something they did not do. For this reason, they may feel that an injustice has been committed against them. Guilty citizens who are searched without probable cause may also suffer penalties even if they are ultimately acquitted, but they will not feel quite as badly that these penalties were not deserved. The injustice can only torment and enrage the truly innocent. Thus we normalize the cost that guilty citizens incur from having their privacy

\textsuperscript{6} Posner [1983] discusses the meaning and value of privacy along these lines.
invaded to zero, and assume that $\eta_I > 0$.

The police incur an additional cost $\eta_P$ if they search innocent citizens without probable cause. The parameter $\eta_P$ is a measure of police accountability. The police are accountable to the people for their mistakes if they suffer a loss, for example in reputation, when they search or arrest innocent citizens without probable cause, which is more likely in a democracy with First Amendment freedoms of speech, press, and assembly. With First Amendment rights, if innocent citizens are wrongfully searched or otherwise mistreated by the police, they can assemble outside police headquarters in peaceful protest without fear of repercussions, or publish scathing articles in leading newspapers without censure, which would reduce police reputation. And if government officials face repeated democratic elections, they are forced to mind their reputation in the eyes of the public, and therefore to discipline police departments, in order to be reelected. Thus in a democracy with First Amendment rights, $\eta_P$ is an increasing function of $\eta_I$, that is, $\eta_P(\eta_I)$, where $\eta_P > 0$.

For now, we assume that indeed $\eta_P > 0$; moreover, to simplify notation, we assume a particular functional form, $\eta_P(\eta_I) = \eta_I = \eta$, which corresponds to strict police accountability. In section 5, we relax these assumptions to analyze the interaction between democracy and the First Amendment, on one hand, and the Fourth Amendment, on the other.

If the police learn $I_\varepsilon$ and choose not to search, then the game is over. If they choose to search, then at time 6, Nature chooses the verdict $v$. The random variable $v$ can be in one of two states, $I_v$, interpreted as the not guilty verdict, or $G_v$, interpreted as the guilty verdict. Define $\alpha_1 = P[I_v|\neg C, G_\varepsilon]$, $\alpha_2 = P[I_v|C, G_\varepsilon]$, $\alpha_3 = P[I_v|\neg C, I_\varepsilon]$, and $\alpha_4 = P[I_v|C, I_\varepsilon]$. Assume that if citizens are innocent and the evidence against them is not probable cause, then the verdict is also always $I_v$, that is, $\alpha_3 = 1$. And if citizens are guilty and the evidence against them is probable cause, then the verdict is always $G_v$, that is, $\alpha_2 = 0$. The important
parameters are $\alpha_1, \alpha_4 \in (0, 1)$.

The Fourth Amendment upholds the right of citizens not to be searched by the police unless the evidence against them is probable cause. In practice, it is an exclusionary rule that indirectly constrains police behavior by making evidence produced by unlawful searches inadmissible at trial, and hence reducing the ultimate conviction probability.

**Definition 1** *The Fourth Amendment is a law that reduces the conviction probability if the police searched a citizen without probable cause. More precisely, it increases $\alpha_4$.***

If citizens are searched without probable cause, and the search does not uncover reliably incriminating evidence, they are acquitted. But if the search uncovers reliably incriminating evidence, they are acquitted only if their lawyers can appeal to the Fourth Amendment. Therefore, in practice, the Fourth Amendment usually results in the acquittal of known criminals. Protecting criminals is not the ultimate objective of the Fourth Amendment, although it is its proximate result. The Fourth Amendment protects the guilty in the hope that “in equilibrium” this will result in fewer innocent citizens being searched by the police. It protects the guilty in order to protect the privacy of the innocent.

The police’s utility depends on the probabilities of the two types of court error (in addition to the search cost). The police’s utility from a rightful conviction or a rightful acquittal is 1, its utility from a wrongful acquittal is 0, and its utility from a wrongful conviction is $U_P = U_P(G_v, \neg C)$, where $U_P \leq U_P(I_v, C) = 0 < U_P(G_v, C) = U_P(I_v, \neg C) = 1$. A citizen’s utility from acquittal is 0 and cost of conviction is $s$ (the sentence length).

Crime, wrongful search, and wrongful conviction are each important components of social welfare. The security, privacy, and freedom of innocent citizens are the basis of a prosperous nation. We study the effects of the Fourth Amendment on these three elements of social welfare, and let readers weigh their relative importance.
4. Equilibrium, Welfare, and Fourth Amendment

Once citizens have learned their benefit from a crime, they each choose whether or not to commit it. Suppose the police search when they do not have probable cause with probability $\sigma_I$. Then if a citizen is of type $w_C$, his payoffs from each of the two strategies are

$$EU_{\text{Citizen}}(-C) = g(\sigma_I) \text{ and } EU_{\text{Citizen}}(C) = w_C + h(\sigma_I) \quad (1)$$

where $g(\sigma_I) = A_1\sigma_I + A_2, \ h(\sigma_I) = A_3(\alpha_4)\sigma_I + A_4$

$$A_1 = -\eta \pi q, \ A_2 = -\pi (1-q)[\eta + s(1-\alpha_1)],$$

$$A_3 = -s\pi (1-q)(1-\alpha_4), \ A_4 = -s\pi q$$

$A_1\sigma_I$ is the probability that police wrongfully search an innocent citizen without probable cause, $\pi q \sigma_I$, times the innocent citizen’s cost of being wrongfully searched without probable cause, consisting of a loss of privacy, $\eta$, but not of a potential wrongful conviction, since by assumption, if innocent citizens are searched without probable cause, they are always acquitted. $A_2$ is the probability that police wrongfully search an innocent citizen with probable cause, $\pi (1-q)$, times the innocent citizen’s cost of being wrongfully searched with probable cause, consisting of a privacy loss, $\eta$, and a potential wrongful conviction, $(1-\alpha_1)s$.

On the other hand, $w_C$ is a citizen’s benefit of committing crime, while $A_3(\alpha_4)\sigma_I + A_4$ is the citizen’s cost of committing crime. $A_3(\alpha_4)\sigma_I$ is the probability that the evidence wrongfully indicates that criminals are innocent but the police search them anyway, $\pi (1-q)\sigma_I$, times the consequent potential cost of conviction, $(1-\alpha_4)s$, which depends on the probability that criminals can be convicted despite having been wrongfully searched by the police, $1-\alpha_4$. $A_4$ is the probability that the evidence rightfully indicates that criminals are guilty and the police come across this evidence, $\pi q$, times the consequent conviction, $s$. 
A citizen of type $w_C$ chooses $\neg C$ if and only if

$$g(\sigma_I) \geq w_C + h(\sigma_I) \Leftrightarrow w_C \leq g(\sigma_I) - h(\sigma_I)$$  \hspace{1cm} (2)

Thus the fraction of citizens who do not commit crime is

$$I(\sigma_I) = F(g(\sigma_I) - h(\sigma_I)) = g(\sigma_I) - h(\sigma_I) = (A_1 - A_3)\sigma_I + (A_2 - A_4)$$  \hspace{1cm} (3)

Since $I(\sigma_I)$ is a probability, it must be between 0 and 1. Since $I(\sigma_I)$ is a monotone function of $\sigma_I$, if $I(\sigma_I)$ is between 0 and 1 at its minimum and at its maximum, then $0 \leq I(\sigma_I) \leq 1$ for any $\sigma_I \in [0,1]$. Therefore, regardless of whether or not $A_1 - A_3 > 0$, $0 \leq I(\sigma_I) \leq 1$ for any $\sigma_I \in [0,1]$ if $0 \leq (A_2 - A_4) \leq 1$ and $0 \leq A_1 - A_3 + (A_2 - A_4) \leq 1$. This condition is sufficient but not necessary; however, since it simplifies the analysis that follows, we focus on parameter ranges where it is satisfied.

Before considering the police’s problem, it is worth noting the effect on crime of an exogenous increase in the police’s search intensity. Taking the derivative of expression (3) with respect to $\sigma_I$ yields $\frac{\partial I(\sigma_I)}{\partial \sigma_I} = A_1 - A_3$. The following is the necessary and sufficient condition for this expression to be positive.

**Proposition 1** An exogenous increase in the police’s search intensity without probable cause reduces crime if and only if $s > \frac{\eta}{(1-\eta)(1-\alpha_2)}$.

Paradoxically, an increase in police search intensity could increase crime. This would happen if, for example, $q \simeq 1$. In this case, if citizens are guilty, the evidence will almost always be probable cause against them, and if they are innocent, the evidence will almost never be probable cause against them. So if the police search more without probable cause, they are only searching the innocent more. This makes it relatively less attractive to be innocent. The same thing happens if $\eta$ is large enough. If the innocent incur a tremendous cost of being searched without probable cause, and they are searched more often without
probable cause, then becoming a criminal becomes relatively more attractive. If one is going
to be treated like a criminal, and the costs of being treated like a criminal even if one is
innocent are very large, then one might as well also derive the benefit from actually being a
criminal. In these cases, an increase in police search intensity causes the borderline innocent
to rebel or turn into criminals.

Suppose the population is distributed according to \((I(\sigma_1), G(\sigma_1))\). If the police observe
\(I_\epsilon\), their expected payoffs from searching \((S)\) and not searching \((\neg S)\) are, respectively,

\[
EU_{Police}(S|I_\epsilon) = \frac{I(\sigma_1)q}{I(\sigma_1)q + G(\sigma_1)(1 - q)}B_1 + \frac{G(\sigma_1)(1 - q)}{I(\sigma_1)q + G(\sigma_1)(1 - q)}B_2(\alpha_4) \tag{4}
\]

\[
EU_{Police}(\neg S|I_\epsilon) = \frac{I(\sigma_1)q}{I(\sigma_1)q + G(\sigma_1)(1 - q)}
\]

where \(B_1 = 1 - \eta - c^S\), \(B_2 = 1 - \alpha_4 - c^S\)

If the signal of probable cause is perfectly accurate \((q = 1)\), the police’s expected utility of not searching when the evidence is not probable cause reaches its maximum of 1. Intuitively, if the police know that evidence that is not probable cause can only come from innocent citizens, then if they observe evidence that is not probable cause, they know for certain that the citizen is innocent, and hence their payoff from not searching the citizen is at the highest possible level.

If the signal of probable cause is perfectly noisy \((q = 1/2)\), the police’s expected utility of not searching when the evidence is not probable cause is exactly equal to the proportion of citizens who are innocent. Intuitively, if the police know that evidence that is not probable cause is as likely to come from guilty citizens as from innocent ones, then if the police observe evidence that is not probable cause, they are completely in the dark as to whether the citizen is innocent, and hence their payoff depends only their prior that the citizen is innocent.

We expect the police to randomize in equilibrium. If all citizens commit crime, the police
want to search them even without probable cause. But if the police search them without probable cause, they do not all want to commit crime. But if enough of them do not commit crime, the police do not want to search them without probable cause, and so on. Let us now locate the parameter ranges where randomization occurs.

**Proposition 2** Let $Z(\sigma_I) := EU_{Police}(S|I_\epsilon) - EU_{Police}(\neg S|I_\epsilon)$, $\overline{Z} := \max\{Z(\sigma_I) : \sigma_I \in [0, 1]\}$, $\underline{Z} := \min\{Z(\sigma_I) : \sigma_I \in [0, 1]\}$, and

$$X := \frac{(1-q)B_2}{(1-q)B_2 + q(1-B_1)}$$

(1) If $\overline{Z} > 0$, $\underline{Z} < 0$, and $A_1 - A_3 > 0$, the game has a unique stable equilibrium

$$(\sigma_I^* = \frac{X - (A_2 - A_4)}{A_1 - A_3}, I(\sigma_I^*) = X)$$

(2) If $\overline{Z} > 0$, $\underline{Z} < 0$, and $A_1 - A_3 < 0$, the game has two stable equilibria, $(\sigma_I^* = 0, I(\sigma_I^*) = A_2 - A_4)$ and $(\sigma_I^* = 1, I(\sigma_I^*) = A_1 - A_3 + A_2 - A_4)$, and one unstable equilibrium

$$(\sigma_I^* = \frac{(A_2 - A_4) - X}{-(A_1 - A_3)}, I(\sigma_I^*) = X).$$

(3) If $\overline{Z} < 0$, the game has a unique stable equilibrium, $(\sigma_I^* = 0, I(\sigma_I^*) = A_2 - A_4)$.

(4) If $\underline{Z} > 0$, the game has a unique stable equilibrium, $(\sigma_I^* = 1, I(\sigma_I^*) = A_1 - A_3 + A_2 - A_4)$.

**Proof.** Proofs of propositions are presented in the Mathematical Appendix. □

The model’s parameter space can be partitioned into six regions, four of which have different equilibrium sets. The six regions are illustrated in Figure 1. The benefits to the police from searching without probable cause depend on the fraction of citizens who commit crime. In Figure 1, the cases where $\overline{Z} > 0$ depict parameter ranges where a large enough fraction of citizens commit crime regardless of the police’s search intensity, that the police always find it optimal to increase their search intensity, which eventually leads them to always search without probable cause ($\sigma_I^* = 1$). Similarly, the cases where $\overline{Z} < 0$ depict parameter ranges where a small enough fraction of citizens commit crime regardless of the police’s
Figure 1: The equilibrium set in the six regions of the model’s parameter space.

search intensity, that the police always find it optimal to decrease their search intensity, which eventually leads them to never search without probable cause ($\sigma^*_i = 0$).

The cases where $\bar{Z} > 0$, $\bar{Z} < 0$ depict parameter ranges where the police’s search intensity affects the fraction of citizens who commit crime to an extent large enough for the police to mind this effect. If $A_1 - A_3 > 0$, an increase in the police’s search intensity reduces the fraction of citizens who commit crime. In these parameter ranges, if the police usually search without probable cause, the fraction of citizens who commit crime is small enough that the police want to reduce their search intensity. Similarly, if the police usually do not search without probable cause, the fraction of citizens who commit crime is large enough that the police want to increase their search intensity. Thus the only stable equilibrium involves the police randomizing between searching and not searching without probable cause.

On the other hand, if $A_1 - A_3 < 0$, an increase in the police’s search intensity without
probable cause increases the fraction of citizens who commit crime. In these (rebellious) ranges, if the police usually search without probable cause, the fraction of citizens who commit crime is large enough that the police want to increase their search intensity even more. If the police usually do not search without probable cause, the fraction of citizens who commit crime is small enough that the police want to reduce their search intensity even further. So the only stable equilibria involve the police either searching always or never.

For now, we focus on the only region of parameter space where the equilibrium is both stable, in the sense that a simple dynamic adjustment process in which the police and citizens take turn myopically best responding to each others’ current strategies converges to it from any strategy pair in its neighborhood, and non-extreme, in the sense that the police’s equilibrium strategy is neither to search with probability 0 nor 1 when the evidence is not probable case (which only happens when sufficiently few or sufficiently many citizens commit crime): \( \overline{Z} > 0, \underline{Z} < 0, \text{ and } A_1 - A_3 > 0. \)

From Proposition 2, we know that in this region the equilibrium is

\[
I(\sigma_t^*) = X = \frac{(1 - q)B_2}{(1 - q)B_2 + q(1 - B_1)} \quad \text{and} \quad \sigma_t^* = \frac{X - (A_2 - A_4)}{A_1 - A_3}
\]  

(5)

In the U.S. (and most other industrialized countries), some search and seizure rules are already at least partially enforced, and the question before policy-makers is whether to strengthen or weaken them at the margin. Taking the derivative of equilibrium search intensity with respect to Fourth Amendment protections yields

\[
\frac{\partial \sigma_t^*}{\partial \alpha_4} = \frac{\frac{\partial X}{\partial \alpha_4}(A_1 - A_3) + \frac{\partial A_4(\alpha_4)}{\partial \alpha_4}[X - (A_2 - A_4)]}{(A_1 - A_3)^2}
\]

(6)

In the numerator of (6), two opposing effects are at play:

- **Direct Effect**: An increase in Fourth Amendment protections directly discourages searches by reducing the probability that they lead to rightful convictions.
• Indirect Effect: An increase in Fourth Amendment protections indirectly encourages searches by directly increasing the crime rate and hence increasing the probability that the searches lead to rightful convictions.

More precisely, the direct effect of the Fourth Amendment corresponds to the term $\frac{\partial X}{\partial \alpha_4}(A_1 - A_3(\alpha_4))$. The term $\frac{\partial X}{\partial \alpha_4}$ is equal to $\frac{-q(1-q)(1-B_1)}{[(1-q)B_2 + q(1-B_1)]}\pi < 0$. Also, in the region of parameter space that we are considering, the only region where the equilibrium is both stable and non-extreme, $A_1 - A_3 > 0$. This “stability condition” guarantees that the direct effect is negative, that is,

$$\frac{\partial X}{\partial \alpha_4}(A_1 - A_3(\alpha_4)) < 0 \quad (7)$$

On the other hand, the indirect effect corresponds to the term $\frac{\partial A_3(\alpha_4)}{\partial \alpha_4}[X - (A_2 - A_4)]$. The term $\frac{\partial A_3(\alpha_4)}{\partial \alpha_4}$ is equal to $s\pi(1-q) > 0$. Also, $X - (A_2 - A_4) > 0$ in the region of parameter space that we are considering. Therefore,

$$\frac{\partial A_3(\alpha_4)}{\partial \alpha_4}[X - (A_2 - A_4)] > 0 \quad (8)$$

The direct effect tends to reduce the police’s equilibrium search intensity without probable cause, whereas the indirect tends to increase it. If the direct effect dominates the indirect effect, then an increase in Fourth Amendment protections reduces the police’s equilibrium search intensity without probable cause. If the indirect effect dominates, the marginal increase in Fourth Amendment protections reduces the police’s search intensity without probable cause. Here is the necessary and sufficient condition for the direct effect to dominate.

**Proposition 3** An increase in Fourth Amendment protections reduces the police’s search intensity without probable cause if and only if

$$X < \bar{X} := \frac{(A_1 - A_3) + s\pi Y(A_2 - A_4)}{(A_1 - A_3) + s\pi Y}$$

where $Y = [(1-q)B_2 + q(1-B_1)]$.  

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Let us now analyze the impact of the Fourth Amendment on three important elements of social welfare, crime, wrongful search, and wrongful conviction.

**Proposition 4** An increase in Fourth Amendment protections increases the crime rate but reduces the probability of wrongful conviction. Moreover, it reduces the probability of wrongful search if \( X < \overline{X} \), but has an ambiguous effect on the probability of wrongful search if \( X > \overline{X} \).

An increase in Fourth Amendment protections reduces the expected cost of crime for citizens. But citizens also expect that the police will increase search intensity without probable cause as they expect there to be more crime (the indirect effect of the Fourth Amendment on search intensity), which increases the expected cost crime. These two effects cancel each other out, leaving only the direct effect of Fourth Amendment protections on police search intensity without probable cause to affect crime. Fourth Amendment protections directly reduce police search intensity without probable cause by reducing the probability that these searches lead to successful conviction, and a reduction in search intensity increases crime. Thus the Fourth Amendment always increases crime, and hence wrongful convictions.

Recently, Atkins and Rubin [2003] found that a time-dummy variable for the 1965 Supreme Court ruling in Mapp v. Ohio, which set the precedent for the Fourth Amendment’s exclusionary rule, had a significantly positive effect on crime, as predicted by our theoretical model.

On the other hand, an increase in Fourth Amendment protections tends to reduce wrongful searches without probable cause by increasing crime, and also tends to increase (reduce) wrongful searches without probable cause by increasing (reducing) police search intensity without probable cause. Therefore, if the Fourth Amendment reduces the police’s search intensity without probable cause (its direct effect on search intensity dominates its indirect effect), then it unambiguously reduces wrongful searches. But if the Fourth Amendment increases the police’s search intensity without probable cause (its indirect effect dominates
its direct effect), then its overall effect on wrongful searches is ambiguous.

Whether the direct effect of the Fourth Amendment’s exclusionary rule dominates its the indirect effect depends on a several parameters. Let us look at the role played by the probability that the evidence comes to the police’s attention, \( \pi \), which increases with the relative size of the police force.

**Proposition 5** An increase in Fourth Amendment protections reduces the police’s search intensity without probable cause for a larger range of parameters the larger is \( \pi \).

If citizens commit crime they are likely to generate evidence that is probable cause. The larger is \( \pi \) the more likely this evidence will come to the police’s attention if one commits crime. Because an increase in \( \pi \) increases the expected cost of crime, the Fourth Amendment does not directly increase crime, and thereby police search intensity, as much the larger is \( \pi \). In other words, an increase in \( \pi \) reduces the indirect effect of the Fourth Amendment on police search intensity, making its direct effect more likely to dominate. Thus, in places heavily guarded with police officers and surveillance cameras, the Fourth Amendment is most likely to reduce the police’s search intensity without probable cause.

5. **Police Accountability**

In the model, we assumed that \( \eta_P(\eta_I) = \eta_I \), that is, the loss in reputation that the police incur from invading the privacy of innocent citizens is directly proportional to the extent to which these citizens value their privacy. This assumption is valid if politicians face repeated democratic elections, and innocent citizens who feel that they have been treated unjustly by the police can assemble outside police headquarters in peaceful protest, or publish their experiences in newspapers, which would reduce police reputation and force politicians to regulate police departments. But if the government has control of the press, and uses this control to suppress dissenting opinions, the police can only suffer minor reputation losses
by invading the privacy of innocent citizens. In this case, $\eta_P < \eta_I$. How does police accountability interact with the Fourth Amendment’s search and seizure law?

**Proposition 6** An increase in Fourth Amendment protections reduces police search intensity without probable cause for a larger parameter range the larger is $\eta_P$ relative to $\eta_I$.

An increase in Fourth Amendment protections reduces the police’s search intensity without probable cause by reducing the probability that such searches lead to successful convictions (the direct effect), but it also directly increases crime. The police would respond to the increase in crime by increasing their search intensity without probable cause (the indirect effect), especially if they did not care too much about searching the innocent in the process. But if the police are wary of wrongfully searching innocent citizens, say because that would harm their reputation, then the indirect effect of the Fourth Amendment on the police’s search intensity without probable cause is smaller. Thus an increase in Fourth Amendment protections is more likely to decrease the police’s search intensity without probable cause, and hence decrease wrongful searches, the more accountable the police are for their mistakes.

It is also worth analyzing the effect of police accountability on the equilibrium behavior of citizens and the police, independently of Fourth Amendment considerations.

**Proposition 7** An increase in $\eta_P$ relative to $\eta_I$ reduces the police’s search intensity without probable cause and increases crime.

Although a marginal increase in police accountability increases crime, it always reduces police search without probable cause, and hence reduces wrongful searches, unlike Fourth Amendment protections, which may or may not reduce wrongful searches. Thus if society’s objective is to protect the privacy of the innocent, it is more likely to achieve its objectives by increasing police accountability than by increasing Fourth Amendment protections. The reason is that an increase in police accountability only reduces police search directly by
reducing the expected utility of search. It also increases crime, but only because it increases
the expected utility of search so that crime must rise in order to make the police indifferent
between searching and not searching, not because it directly increases the expected utility
of crime. In contrast, Fourth Amendment protections directly increase the expected utility
of crime, and therefore tend to increase police search indirectly, in addition to tending to
reduce it directly by reducing the expected utility of search. However, as we will see in the
next section, both police accountability and Fourth Amendment protections are needed for
progress in search technology to lead to a Utopian equilibrium in which the police never
search without probable cause and crime is very low.

6. 2084

Thus far, we have only analyzed the region of the model’s parameter space wherein the equi-
librium is stable and non-extreme, in the sense that the police do not search with probability
0 or 1 without probable cause. Let us call this region \(R_{2003}\), because it is the only region
that gives rise to an equilibrium that resembles the state of interaction between citizens and
the police in today’s world. But how will citizens and the police interact in the future, when
technological advances might permit the police to monitor citizens all the time?

The U.S. government already maintains surveillance satellites in space that are equipped
with microwave technology that gathers huge data sets, which computer imaging systems
then transform into visual images. From space, the satellites can “see” objects as small as
three feet across through clouds and in darkness. This technology creates the possibility
that the government will soon be able to gain detailed visual images of private dwellings
without installing a camera or entering the premises. [Greenfield, 1991]

The U.S. Transportation Security Administration uses voice stress analysis and biometric
technologies to prevent a person who might pose a danger to air safety from boarding an airplane. Characteristics such as fingerprints, hand geometry, facial appearance, and retina and iris scans are all considered biometric measures. But biometrics have the potential to reveal much more about a person than simply their identity. For instance, retina scans can reveal medical conditions such as pregnancy, high blood pressure, and AIDS [see Star, 2002].

DNA forensic technology has become the most effective method of identifying criminals. An important question currently before the Supreme Court is whether to require probable cause before sampling the DNA of suspects prior to their arrest by piercing their skin or scraping the inside of their mouth [see Jewkes, 2001]. Technological advances have also made it possible to collect DNA evidence for future use. Many law enforcement agencies already maintain databanks containing the genetic information of convicted felons.

In the future, biometric measures, wiretaps, X-rays, and DNA testing will become increasingly accurate, omniscient, and invasive. In terms of the model, $\alpha_1$ will increase because search technologies will become more accurate, and $\pi$ will increase because they will become more omniscient. But because they will also become more invasive, they will increase the individual’s cost of privacy invasion, $\eta_I$. If the government had technology that were capable of continuously monitoring every citizen’s actions, medical conditions, genetic predispositions, and perhaps even thoughts, and if they used these technologies to search citizens even without probable cause, then citizens would incur a great loss of privacy, $\eta_I$.

In the model, if $\eta_I$ will increase enough, then all else constant, $A_1 - A_3$ will become negative. Thus advances in search technologies would propel the world from $R_{2003}$ where $A_1 - A_3 > 0$, to another region of parameter space where $A_1 - A_3 < 0$. We assume that as search technologies become more advanced, $\alpha_1$ will tend to 1, $\pi$ will tend to 1, and $\eta_I$ will increase enough to ensure that $A_1 - A_3 < 0$. There are three regions of parameter space
where $A_1 - A_3 < 0$, each with a different equilibrium set. In the first region, call it $R_{2084}^1$, there is a unique, stable, pure-strategy equilibrium in which the police always search without probable cause, and a great many citizens commit crime. This is an anti-Utopian outcome.

**Definition 2** An anti-Utopian equilibrium is one in which the police always search without probable cause and most citizens commit crime.

In the second region, call it $R_{2084}^2$, there is a unique, stable, pure-strategy equilibrium in which the police never search without probable cause, and very few citizens commit crime. This is a Utopian outcome.

**Definition 3** A Utopian equilibrium is one in which the police never search without probable cause and most citizens do not commit crime.

In the third region, call it $R_{2084}^3$, there are two stable, pure-strategy equilibria, the anti-Utopian equilibrium that arises in $R_{2084}^1$ and the Utopian equilibrium in $R_{2084}^2$, and a single, unstable, mixed-strategy equilibrium. By increasing citizens’ cost of privacy invasion, advances in search technologies could therefore alter the way citizens and the police interact in several different ways, depending on parameters.

Several science fiction writers have attempted to predict which way technological progress will alter the interaction between citizens and the government. In his novel *1984*, Orwell depicts a dark future in which the police employ technology capable of monitoring every citizen’s actions to further the government’s totalitarian objectives. One of the search technologies employed by Big Brother’s government to monitor the population is the telescreen, a large rectangular mirror-like plaque that is attached to a wall in almost every apartment in the land. The ubiquitous telescreen not only transmits but also receives information from within its range. The government is also conducting intensive research on the problem of finding out what people are thinking against their will, because the Party cannot stand that even one individual, such as Winston Smith, can have a heterodox thought.
Through the telescreen and the Thought Police, the government is relentlessly searching all citizens, and so most of the time, it is searching them without probable cause. But are many citizens rebelling and turning into criminals, as our model predicts? Although Winston knows that writing in a diary is forbidden, he eventually ends up doing it, and although Winston and Julia know that making love in the country-side is forbidden, they eventually end up doing it, because when privacy is so scarce, a mere moment of it is priceless. In Orwell’s anti-Utopia, the police search without probable cause relentlessly, and all, even mildly, dissenting citizens eventually rebel and commit crime, only inevitably to be caught and punished. This approximates the outcome in region $R_{2084}^1$ in the model.

In Orwell’s 1984, technological change has ultimately lead to anti-Utopia. But it could also have lead to Utopia. Is there a key factor that might guarantee that it leads to anti-Utopia? Orwell describes a government that has complete control over the press and the media. The telescreen only shouts out Party slogans all day. Citizens can turn the volume of the telescreen down, but never off. Books are written by the Ministry of Truth, a branch of government. Revolutionary books are banned. History is rewritten to conform with the ideologies of the Party. Big Brother’s government is not accountable to the people.

Returning to our simple model, we now demonstrate that absence of police accountability leads to the anti-Utopian equilibrium.

**Proposition 8** (The Big Brother Theorem, Part I). If the police are not accountable to the people, progress in search technology will lead to the anti-Utopian equilibrium.

Absence of police accountability is sufficient for anti-Utopia. If the police are not accountable, they will suffer few losses from searching the innocent without probable cause. So they will search with greater intensity without probable cause. But progress in search technology will make these searches more invasive, so that the world will be in a rebellious (mutual trust) region of the model’s parameter space, where high (low) search rates are associated
with high (low) crime rates. If innocent citizens have to suffer the costs of being treated like criminals, and these costs are large, then they will find it in their interest to at least derive the benefits from actually being criminals. For this reason, more innocent citizens will rebel and commit crime. But this increase in crime will push the police to increase their search intensity even further, and so on until the police will always search without probable cause and a great many citizens will commit crime—the anti-Utopian equilibrium.

On the other hand, absence of Fourth Amendment protections is neither necessary nor sufficient for progress in search technology to lead to anti-Utopia. To understand the role of the Fourth Amendment in an Orwellian future, we first prove a lemma.

**Lemma 1** In $R^3_{2084}$, the Utopian equilibrium risk dominates and yields higher social surplus than the anti-Utopian equilibrium if strong Fourth Amendment protections are in place.

In $R^2_{2084}$, the Utopian and anti-Utopian equilibria are strict, and hence stable, but the former risk dominates the latter if strong Fourth Amendment protections are in place. Henceforth, we assume that the Utopian equilibrium is the one that emerges in $R^3_{2084}$ given strong Fourth Amendment protections.\(^7\) With this assumption, the following result can be proved.

**Proposition 9** (*The Big Brother Theorem, Part II*). If the police are accountable to the people and strong Fourth Amendment protections are enforced by the courts, progress in search technology will lead to the Utopian equilibrium.

Strong Fourth Amendment protections and strict police accountability are jointly sufficient for Utopia. In combination, they will ensure that police search intensity without probable cause is very low. But search technology will become increasingly invasive, so that citizens and the police will be in a mutual trust (rebellious) region of the model’s parameter

\(^7\) An argument in favor of the risk dominant equilibrium is set forth in Carlsson and van Damme [1993], who analyze equilibrium selection in what they call a “global game,” which is a game where Nature selects the payoff structure from a given class of games and each player only receives a noisy signal of the game selected. They show that in $2 \times 2$ games, when the noise vanishes, iterated elimination of dominated strategies in the global game forces players to coordinate on the risk dominant equilibrium.
space, where low (high) search rates are associated with low (high) crime rates. In such a region, if the police show trust toward citizens by respecting their privacy and freedom, then citizens thrive on their freedom to such an extent that they will not want to abuse of it. Thus with police accountability and Fourth Amendment protections lowering the search intensity, the crime rate will be lower. Because the crime rate will be lower, the police will reduce their search intensity even further, and so on until the police never search without probable cause and most citizens do not commit crime—the Utopian equilibrium.  

7. Conclusion

The model’s principal implications may be summarized as follows:

1. A decrease in Fourth Amendment protections tends to increase the police’s search intensity directly by increasing the probability that search leads to successful conviction, and tends to decrease the police’s search intensity indirectly by decreasing crime.

2. If the indirect effect dominates, a decrease in Fourth Amendment protections decreases police search intensity, and has an ambiguous effect on wrongful searches.

3. If the direct effect dominates, a decrease in Fourth Amendment protections increases police search intensity and increases wrongful searches.

An implicit assumption in the Big Brother Theorem (like in Orwell’s novel) is that the police have better control of emerging technology than the population. Otherwise criminals might have new hiding technology to counter the police’s new search technology. However, the Big Brother Theorem continues to hold even if criminals also have new hiding technology, as long as the innocent do not. If the innocent also have new hiding technology to counter the police’s new search technology, and thereby protect their privacy, then technological progress would not increase $\eta_I$ enough to propel the interaction to a region where $A_1 - A_3 < 0$. In this case, the interaction would remain in a region where $A_1 - A_3 > 0$. From Figure 1, we know that there are three such regions, a first one where $Z < 0$ and $\overline{Z} > 0$ and the unique stable equilibrium is mixed, a second one where $\overline{Z} > 0$ and the unique stable equilibrium is anti-Utopian, and a third one where $\overline{Z} < 0$ and the unique stable equilibrium is Utopian. Progress in search technology, if it does not increase $\eta_I$ sufficiently to propel the interaction to a region where $A_1 - A_3 < 0$, would single-handedly determine where the interaction ends up among the three regions where $A_1 - A_3 > 0$ by increasing $\alpha_1$ and $\pi$. An increase in $\alpha_1$ increases $I(\sigma_I)$ and thus lowers $Z$ and $\overline{Z}$. Therefore, the entire parameter range moves down toward the Utopian region. Similarly, an increase in $\pi$ increases $I(\sigma_I)$ and thus the police’s expected utility of search decreases, which moves $\overline{Z}$ and $\overline{Z}$ down toward the Utopian region also.
4. A decrease in Fourth Amendment protections is more likely to increase police search intensity and wrongful searches

(a) the larger is the police force relative to the total population

(b) the more accountable are the police to the people.

5. A decrease in Fourth Amendment protections reduces crime but increases wrongful convictions.

6. A decrease in police accountability reduces crime, and increases wrongful searches as well as wrongful convictions.

7. If the police are not accountable, long-term technological progress will lead to an outcome in which the police always search without probable cause and many citizens commit crime.

8. If the police are accountable and Fourth Amendment protections are enforced, technological progress will lead to an outcome in which the police never search without probable cause and few citizens commit crime.

Implications (1) through (4) could serve as a guide for future empirical research on the effects of the Fourth Amendment on police search practices. The existing literature is notoriously confusing. Oaks [1970] found that the Fourth Amendment’s exclusionary rule (as measured by a time-dummy variable for the 1965 Supreme Court ruling in Mapp v. Ohio) had no significant effect on arrests by the police in Cincinnati. Cannon [1974] replicated Oaks’s Cincinnati research in thirteen other cities and showed that the effect of the exclusionary rule in Cincinnati was not typical. In several other cities the exclusionary rule significantly reduced the number of arrests. Based on these studies, the Supreme Court reluctantly concluded, in U.S. v. Janis [428 US 433, 1976], that “No empirical researcher,
proponent or opponent [of the exclusionary rule] has yet been able to establish with any assurance whether the rule has a deterrent effect [on searches by the police].” More recently some researchers have used interviews with individual officers [Wasby, 1976, Krantz et al, 1979, Orfield, 1987, Canon, 1991], and others have used field observation [Skolnick, 1994, Gould, 2000], as an alternative to official records. The results of these studies are also mixed, with some studies finding a significant effect in some parts of the country, and other studies finding no significant effect in other parts of the country. Thus researchers in the field still openly admit that the extant empirical research neither proves nor disproves the inhibitory effect of the exclusionary rule.

The model developed in this paper offers a possible explanation for this apparent confusion. As implications (1) through (4) suggest, the Fourth Amendment’s exclusionary rule could have qualitatively different effects on police search behavior if the values of the parameters are sufficiently different. The parameters in the model may have different values in different parts of the country. For example, (4.a) suggests that the Fourth Amendment is less likely to inhibit police search in Houston, where the mean number of sworn officers per 100,000 citizens from 1970 to 1992 was one of the lowest in the nation among big cities at 265, than in Chicago, where mean officers per 100,000 citizens during this period was the second highest in the nation at 475 [Levitt, 1997]. Implication (4.b) suggests the Fourth Amendment is more likely to inhibit police search in states or cities where the police are more accountable. One way to measure police accountability at the city level is through a dummy variable indicating whether the city’s police chief is elected or appointed. Controlling for these factors may improve the quality of future empirical research on the effects of the Fourth Amendment on police search behavior.
Moreover, existing studies have only estimated the direct effect of the exclusionary rule on police search practices, ignoring its indirect effect through crime. Our theoretical model suggests that police search and crime are determined simultaneously. Future empirical research should look at the effect of the exclusionary rule, and other important laws, on crime and police searches (or arrests) estimated simultaneously.

A Mathematical Appendix

Proof of Proposition 2. We know that

\[
Z(\sigma_I) = EU_{Police}(S|I_\varepsilon) - EU_{Police}(\neg S|I_\varepsilon) \\
= I(\sigma_I)[q(B_1 - 1) - (1 - q)B_2(\alpha_4)] + (1 - q)B_2(\alpha_4)
\]

When \(Z(\sigma_I) > 0\), \(EU_{Police}(S|I_\varepsilon) > EU_{Police}(\neg S|I_\varepsilon)\), and vice versa.

Case 1: \(Z > 0\), \(\underline{Z} < 0\), and \(A_1 - A_3 > 0\). If \(A_1 - A_3 > 0\), \(I(\sigma_I)\) increases as \(\sigma_I\) increases. Since \([q(B_1 - 1) - (1 - q)B_2] < 0\), \(Z(\sigma_I)\) is a decreasing function of \(\sigma_I\). That is, \(Z(\sigma_I)\) is at its maximum when \(\sigma_I = 0\), and at its minimum when \(\sigma_I = 1\). Thus the conditions that \(Z > 0\) and \(\underline{Z} < 0\) mean that \(Z(\sigma_I) > 0\) at \(\sigma_I = 0\) and \(Z(\sigma_I) < 0\) at \(\sigma_I = 1\). Since \(EU_{Police}(S|I_\varepsilon) > EU_{Police}(\neg S|I_\varepsilon)\) when \(\sigma_I = 0\), but \(EU_{Police}(S|I_\varepsilon) < EU_{Police}(\neg S|I_\varepsilon)\) when \(\sigma_I = 1\), no pure-strategy equilibrium exists. The unique mixed-strategy equilibrium is defined by

\[Z(\sigma_I) := I(\sigma_I)q(B_1 - 1) + G(\sigma_I)(1 - q)B_2 = 0.\]

Thus

\[I(\sigma_I^*) = \frac{(1 - q)B_2}{(1 - q)B_2 + q(1 - B_1)} = X\]

Therefore, the police’s equilibrium search intensity without probable cause is

\[\sigma_I^* = \frac{X - (A_2 - A_4)}{(A_1 - A_3)}\]

This mixed-strategy equilibrium is stable. Suppose police perturb their strategy by \(\varepsilon > 0\). At \(\sigma_I^* + \varepsilon\), \(Z(\sigma_I = 0) < 0 \Leftrightarrow EU_{Police}(S|I_\varepsilon) < EU_{Police}(\neg S|I_\varepsilon)\) since \(Z(\sigma_I)\) is a decreasing function of \(\sigma_I\).
That is, police would want to decrease their search intensity as a result. Any small perturbation from the equilibrium would eventually lead back to the original equilibrium. The condition that $Z(\sigma_I)|_{\sigma_I=0} > 0, Z(\sigma_I)|_{\sigma_I=1} < 0$ is equivalent to $(A_2 - A_4) < X < A_1 - A_3 + (A_2 - A_4)$.

Case 2: $\overline{Z} > 0$, $\underline{Z} < 0$, and $A_1 - A_3 < 0$. If $A_1 - A_3 < 0$, $I(\sigma_I)$ decreases as $\sigma_I$ increases. Given that $[q(B_1 - 1) - (1 - q)B_2] < 0$, $Z(\sigma_I)$ is an increasing function of $\sigma_I$. That is, $Z(\sigma_I)$ is at its minimum when $\sigma_I = 0$, and at its maximum when $\sigma_I = 1$. Hence, the conditions that $\overline{Z} > 0$ and $\underline{Z} < 0$ mean that $Z(\sigma_I) > 0$ at $\sigma_I = 1$ and $Z(\sigma_I) < 0$ at $\sigma_I = 0$. Since $EU_{Police}(S|I_\varepsilon) > EU_{Police}(\neg S|I_\varepsilon)$ when $\sigma_I = 1$, but $EU_{Police}(S|I_\varepsilon) < EU_{Police}(\neg S|I_\varepsilon)$ when $\sigma_I = 0$, the two pure-strategy equilibria are: $(\sigma_I^* = 0, I(\sigma_I^* = 0))$ and $(\sigma_I^* = 1, I(\sigma_I^* = 1))$. The unique mixed equilibrium is defined by $Z(\sigma_I) := I(\sigma_I)q(B_1 - 1) + G(\sigma_I)(1 - q)B_2 = 0$. Thus

$$I(\sigma_I^*) = \frac{(1 - q)B_2}{(1 - q)B_2 + q(1 - B_1)} = X$$

Therefore, the police’s equilibrium search intensity without probable cause is

$$\sigma_I^* = \frac{(A_2 - A_4) - X}{-(A_1 - A_3)}$$

This mixed-strategy equilibrium is not stable. Suppose police increase their search intensity by $\varepsilon$. For $\sigma_I^* + \varepsilon$, $EU_{Police}(S|I_\varepsilon) > EU_{Police}(\neg S|I_\varepsilon)$ since $Z(\sigma_I)$ is an increasing function of $\sigma_I$. In this case, police would want to increase search intensity even more, and vice versa. The condition that $Z(\sigma_I)|_{\sigma_I=0} < 0, Z(\sigma_I)|_{\sigma_I=1} > 0$ is equivalent to $0 < A_1 - A_3 + (A_2 - A_4) < X < (A_2 - A_4)$.

Case 3: $\overline{Z} < 0$. Regardless of $A_1 - A_3 > 0$, the condition that $\overline{Z} < 0$ implies $EU_{Police}(S|I_\varepsilon) < EU_{Police}(\neg S|I_\varepsilon)$ for all $\sigma_I \in [0, 1]$. Therefore, the unique pure-strategy equilibrium is $\sigma_I^* = 0$.

Case 4: $\underline{Z} > 0$. Regardless of $A_1 - A_3 > 0$, the condition that $\underline{Z} > 0$ implies $EU_{Police}(S|I_\varepsilon) > EU_{Police}(\neg S|I_\varepsilon)$ for all $\sigma_I \in [0, 1]$. Therefore, the unique pure-strategy equilibrium is $\sigma_I^* = 1$. 

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Proof of Proposition 3. Comparing (7) and (8) in the text, we find that

\[-\frac{\partial X}{\partial \alpha_4} (A_1 - A_3(\alpha_4)) > \frac{\partial A_3(\alpha_4)}{\partial \alpha_4} [X - (A_2 - A_4)]\]

\[\Leftrightarrow \frac{1 - X}{[(1-q)B_2 + q(1-B_1)]} > s\pi \left( \frac{X - (A_2 - A_4)}{A_1 - A_3} \right) \Leftrightarrow X < \bar{X} := \frac{A_1 - A_3 + s\pi Y(A_2 - A_4)}{A_1 - A_3 + s\pi Y}\]

where \(Y = (1-q)B_2 + q(1-B_1)\).

Proof of Proposition 4. The equilibrium crime rate is \(1 - I(\sigma^*_I)\), where \(I(\sigma^*_I) = (A_1 - A_3)\sigma^*_I + (A_2 - A_4)\). Differentiating with respect to \(\alpha_4\) yields

\[\frac{\partial (1 - I(\sigma^*_I))}{\partial \alpha_4} = \frac{\partial A_3}{\partial \alpha_4} \sigma^*_I - (A_1 - A_3)\frac{\partial \sigma^*_I}{\partial \alpha_4} = \frac{\partial A_3}{\partial \alpha_4} \sigma^*_I - \frac{\partial X}{\partial \alpha_4} \frac{\partial A_3}{\partial \alpha_4} [X - (A_2 - A_4)]\]

\[= -\frac{\partial X}{\partial \alpha_4} > 0\]

The probability of a wrongful conviction is \(P[G_v, \neg C] = I(\sigma^*_I)(1-q)\pi(1-\alpha_1)\). Differentiating with respect to \(\alpha_4\) yields

\[\frac{\partial P[G_v, \neg C]}{\partial \alpha_4} = \frac{\partial I(\sigma^*_I)}{\partial \alpha_4} (1-q)\pi(1-\alpha_1) < 0\]

The probability of a wrongful search is \(P[S, \neg C] = I(\sigma^*_I)q\pi\sigma_I + I(\sigma^*_I)(1-q)\pi\). Differentiating with respect to \(\alpha_4\) and using the chain rule twice yields

\[\frac{\partial P[S, \neg C]}{\partial \alpha_4} = \frac{\partial I(\sigma^*_I)}{\partial \alpha_4} [q\pi\sigma_I + (1-q)\pi] + \frac{\partial \sigma^*_I}{\partial \alpha_4} I(\sigma^*_I)\]

We know \(\frac{\partial I(\sigma^*_I)}{\partial \alpha_4} < 0\). If \(X < \bar{X}, \frac{\partial \sigma_I}{\partial \alpha_4} < 0\), so \(\frac{\partial P[S, \neg C]}{\partial \alpha_4} < 0\). If \(X > \bar{X}, \frac{\partial \sigma_I}{\partial \alpha_4} > 0\), so \(\frac{\partial P[S, \neg C]}{\partial \alpha_4} \geq 0\).

Proof of Proposition 5. From the proof of Proposition 3 above, we know that the direct effect is greater than the indirect effect if and only if

\[\frac{1 - X}{[(1-q)B_2 + q(1-B_1)]} > \frac{s\pi}{s(1-q)(1-\alpha_4) - \eta q} \left( \frac{X - \pi [sq - s(1-q)(1-\alpha_1) - \eta(1-q)]}{A_1 - A_3} \right)\]

\[\Leftrightarrow \frac{1 - X}{[(1-q)B_2 + q(1-B_1)]} > s \left( \frac{X - \pi [sq - s(1-q)(1-\alpha_1) - \eta(1-q)]}{s(1-q)(1-\alpha_4) - \eta q} \right)\]
The left-hand-side is unaffected by the change in $\pi$, while the right-hand-side is decreasing as $\pi$
increases. Thus as $\pi$ increases, the condition that the direct effect is greater than the indirect
effect is more likely to be satisfied.

**Proof of Proposition 6.** $A_3$, $A_4$, and $B_2$ do not depend on $\eta_P$ or $\eta_I$. In terms of $\eta_P$ and
$\eta_I$, $A_1 = -\eta_I \pi q$, $A_2 = -\pi (1 - q) [\eta_I + s(1 - \alpha_1)]$, and $B_1 = 1 - \eta_P - c^S$. Therefore, only $B_1$
depends on $\eta_P$. Since $X$ is a function of $B_1$, it is also a function of $\eta_P$. From expression (6) in the
text, we know that

$$\frac{\partial \sigma_I^*}{\partial \alpha_4} = \frac{- (1-q) (1-X(\eta_P))}{(1-q)B_2 + (1-B_1(\eta_P))} (A_1 - A_3) + \frac{\partial A_3(\alpha_4)}{\partial \alpha_4} [X(\eta_P) - (A_2 - A_4)]$$

$$\implies \frac{\partial^2 \sigma_I^*}{\partial \alpha_4 \partial \eta_P} = \frac{(1-q)q}{[(1-q)B_2 + (1-B_1(\eta_P))\alpha_4]^2} [(A_1 - A_3)q(1 - 2X) - s\pi(1 - q)B_2]$$

Suppose the Fourth Amendment’s indirect effect dominates its direct effect, so that

$$- \frac{\partial X}{\partial \alpha_4} (A_1 - A_3) < \frac{\partial A_3(\alpha_4)}{\partial \alpha_4} [X - (A_2 - A_4)] \implies - \frac{\partial X}{\partial \alpha_4} (A_1 - A_3) < \frac{\partial A_3(\alpha_4)}{\partial \alpha_4} X$$

$$\implies (A_1 - A_3)(1 - X) < s\pi(1 - q)B_2 \implies q(A_1 - A_3)(1 - X) < s\pi(1 - q)B_2$$

In this case, $\frac{\partial^2 \sigma_I^*}{\partial \alpha_4 \partial \eta_P} < 0$. In other words, an increase in $\eta_P$ reduces the range of parameters where
the direct effect dominates.

**Proof of Proposition 7.** From Proposition 2, we know that in equilibrium

$$\sigma_I^* = \frac{X - (A_2 - A_4)}{A_1 - A_3} \quad \text{and} \quad I(\sigma_I^*) = X = \frac{B_2(1-q)}{(1-q)B_2 + q(1-B_1(\eta_P))}$$

But $\partial X/\partial \eta_P < 0$. Thus the crime rate is higher, and the police’s search intensity without probable
cause is lower, if the police are accountable to the public for their errors than if they are not.

**Proof of Proposition 8.** The current state of interaction between citizens and the police
is given by the unique stable mixed-strategy equilibrium, which only arises in $R_{2003}$, defined by

$$(A_2 - A_4) < X < (A_1 - A_3) + (A_2 - A_4)$$

where $A_1 - A_3 > 0$. By assumption, in the future, $\eta_I$ will increase sufficiently to propel the world to a region in which $A_1 - A_3 < 0$. In this region,
\[(A_2 - A_4) > (A_1 - A_3) + (A_2 - A_4) \geq 0,\] where the last inequality holds by assumption. Now assume that the police are not accountable in a dynamic sense: \(\eta_p\) does not increase as \(\eta_I\) increases. Note that as \(\eta_I\) increases, \(X\) is unaffected; indeed, \(X\) depends, not on \(\eta_I\), but on \(\eta_p\), which does not change by assumption. We know that \(A_2 - A_4 = -\eta_I\pi(1 - q) - s\pi(1 - q)(1 - \alpha_1) + s\pi q \geq 0.\) Therefore, as \(\eta_I\) increases, \(A_2 - A_4\) decreases, and \((A_1 - A_3) + (A_2 - A_4)\) decreases, which necessarily propels the world to \(R_{2084}\), defined by \((A_1 - A_3) + (A_2 - A_4) < (A_2 - A_4) < X.\) In this region, the unique, stable equilibrium is the anti-Utopian one. Note that the world cannot end up in either \(R_{2084}^2\) or \(R_{2084}^3\), since \(R_{2084}^2\) corresponds to the parameter range that satisfies \(X < (A_1 - A_3) + (A_2 - A_4) < (A_2 - A_4)\) and \(R_{2084}^3\) corresponds to the parameter range \((A_1 - A_3) + (A_2 - A_4) < X < (A_2 - A_4).\)

**Proof of Lemma 1.** The citizens’ surplus in the Utopian equilibrium is

\[
U_{\text{Citizens}}^{Utopia} = \int_{0}^{(A_2 - A_4)} A_2 dw_C + \int_{(A_2 - A_4)}^{1} (w_C + A_4) dw_C
\]

\[
= A_2 (A_2 - A_4) + \left\{ (1 - (A_2 - A_4)) \left[ \frac{1}{2} (1 + (A_2 + A_4)) \right] \right\}
\]

The citizens’ surplus in the anti-Utopian equilibrium is

\[
U_{\text{Citizens}}^{Anti-Utopia} = \int_{0}^{A_1 - A_3 + A_2 - A_4} (A_1 + A_2) dw_C + \int_{A_1 - A_3 + A_2 - A_4}^{1} (w_C + A_3 + A_4) dw_C
\]

\[
= (A_1 + A_2)(A_1 - A_3 + A_2 - A_4)
\]

\[
+ \left\{ (1 - (A_1 - A_3 + A_2 - A_4)) \left[ \frac{1}{2} (1 + (A_1 + A_3 + A_2 + A_4)) \right] \right\}
\]

Therefore, \(U_{\text{Citizens}}^{Utopia} - U_{\text{Citizens}}^{Anti-Utopia} = -(2A_3)[1 - (A_2 - A_4)] - 2A_1(A_1 - A_3 + A_2 - A_4) + (A_1 - A_3)(A_1 + A_3) > 0.\) On the other hand, the police’s surplus in the Utopian equilibrium is

\[
U_{\text{Police}}^{Utopia} = \frac{I(\sigma_I^*)q}{I(\sigma_I^*)q + [1 - I(\sigma_I^*)](1 - q)} = \frac{(A_2 - A_4)q}{(A_2 - A_4)q + (1 - (A_2 - A_4))(1 - q)}
\]

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The police’s surplus in the anti-Utopian equilibrium is

\[
U_{\text{Police, Anti-Utopia}} = \frac{I(\sigma_I^*)qB_1 + (1 - I(\sigma_I^*))q(1 - q)B_2}{I(\sigma_I^*)q + [1 - I(\sigma_I^*)](1 - q)}
\]

\[
= \frac{[A_1 - A_3 + A_2 - A_4]qB_1 + [1 - (A_1 - A_3) - (A_2 - A_4)]q(1 - q)B_2}{[A_1 - A_3 + A_2 - A_4]q + [1 - (A_1 - A_3) - (A_2 - A_4)](1 - q)}
\]

As \(\alpha_4 \to 1\), \(B_2 \to 0\), so that \(U_{\text{Police, Anti-Utopia}} < 0\). But \(U_{\text{Police, Utopia}} > 0\) for all parameters. Therefore, the Utopian equilibrium generates higher social surplus, the sum of citizens’ surplus and the police’s surplus, than the anti-Utopian equilibrium. Now let \(I\) denote the probability that the representative citizen will not commit crime. Then the police’s utility from searching with or without probable cause is \(U_{\text{Police}}(S|I) = (1 - I)(B_2 + \gamma) + I(B_1 + \delta)\), where \(\gamma\) and \(\delta\) are some function of parameters. Similarly, the police’s utility from not searching with or without probable cause is \(U_{\text{Police}}(\neg S|I) = (1 - I)(\gamma) + I(1 + \delta)\). Therefore, \(U_{\text{Police}}(S|I) < U_{\text{Police}}(\neg S|I) \iff (1 - I)B_2 < I(1 - B_1)\). Now, \(B_2 \to 0\) as \(\alpha_4 \to 1\), and \(1 - B_1\) is an increasingly large positive number as \(\eta\) increases, so for almost all \(I\), \(U_{\text{Police}}(S|I) < U_{\text{Police}}(\neg S|I)\). Thus \(\sigma_I^* = 0\) risk dominates \(\sigma_I^* = 1\) for the police.

**Proof of Proposition 9.** Suppose the police are accountable, so that \(\eta_P\) increases with \(\eta_I\). For notational simplicity, consider the case where \(\eta_P = \eta_I = \eta\). In this case, as \(\eta\) increases, \(X\) also decreases. Let \(\widehat{\eta}\) denote the value of \(\eta\) satisfying \(A_1(\eta) - A_3 = 0\), \(\widehat{\eta} = \frac{s(1-q)(1-\alpha_4)}{q}\).

For any \(\eta > \widehat{\eta}\), \((A_1(\eta) - A_3) + (A_2(\eta) - A_4) < (A_2(\eta) - A_4)\). As \(\eta \to \widehat{\eta}\) from the left, \((A_1(\eta) - A_3) + (A_2(\eta) - A_4) \to (A_2(\eta) - A_4)\). If \(X(\eta) > (A_2(\eta) - A_4)\) as \(\eta \to \widehat{\eta}\), then the only region that can be reached is \(R_{2084}^1\). If \(X(\eta) < (A_2(\eta) - A_4)\) as \(\eta \to \widehat{\eta}\), then \(R_{2084}^2\) or \(R_{2084}^3\) may be reached. By assumption, \(\alpha_1 \to 1\) and \(\pi \to 1\). Therefore, as \(\eta \to \widehat{\eta}\),

\[
X|_{\eta \to \widehat{\eta}} = \frac{(1-q)(1-\alpha_4-c^S)}{(1-q)(1-\alpha_4-c^S)+s(1-q)(1-\alpha_4)+c^Sq}
\]

\[
(A_2 - A_4)|_{\eta \to \widehat{\eta}} = s\left(q - \frac{(1-q)^2}{q}(1-\alpha_4)\right)
\]
An increase in Fourth Amendment protections decreases $X|_{\eta \to \tilde{\eta}}$, that is,

$$\frac{\partial X|_{\eta \to \tilde{\eta}}}{\partial \alpha_4} = -c^2(1-q) \left[ (1-q)(s+1) + (2q-1) \right] \left[ (1-q)B_2 + q(1-B_1) \right]^2 < 0$$

If Fourth Amendment protections are strong, that is $\alpha_4 \to 1$, then $X|_{\eta \to \tilde{\eta}} < (A_2 - A_4)|_{\eta \to \tilde{\eta}} \iff \frac{-(1-q)}{q} < sq$. The expression on the right hand side of the inequality is negative, since $q > 1/2$ by assumption, while the expression on the right hand side is positive. Therefore, the inequality is always satisfied. This means that either $R^2_{2084}$ or $R^3_{2084}$ is reached as $\eta$ increases. Moreover, by Lemma 1, the risk dominant equilibrium in both these regions is Utopian. Therefore, assuming the risk dominant equilibrium is chosen, police accountability and strong Fourth Amendment protections are jointly sufficient for Utopia.

REFERENCES


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