Asymmetric Information, Pretrial Negotiation and Optimal Decoupling

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Abstract

The "decoupled" liability system awards the plaintiff an amount that differs from what the defendant pays. The current approach to the optimal decoupling design is based on the assumption of complete information, which results in an optimal liability for the defendant "as much as he can afford." This extreme conclusion may hinder the acceptability of the decoupling system. This paper proposes an alternative design based on the assumption that agents in the post-accident subgame have asymmetric information. Our model indicates that the optimal penalty faced by the defendant is generally greater than the optimal award to the plaintiff. When the potential harm is sufficiently large, the optimal penalty can be approximated by a multiple of the harm, but the plaintiff receives only a finite amount of the damages regardless of the loss suffered. Such a decoupling scheme deters frivolous lawsuits without reducing the defendants' incentives to exercise care. Additionally, this paper derives comparative static results concerning how the trial costs of the plaintiff and defendant affect the optimal design of decoupling.

1 Introduction

In their seminal paper in this *Journal*, Polinsky and Che (1991) demonstrate the efficiency of a "decoupled" liability regime. In that system, the plaintiff is awarded an amount different from (and usually smaller than) what the defendant pays.[1] The original idea of decoupling liability was proposed by Schwartz (1980) and Salop and White (1986), and was partly motivated to reduce excessive liability, mainly due to punitive damages, faced by businesses. By reducing the portion received by the plaintiff without increasing the payment by the defendant, the latter's excessive liability may be reduced.[2] The contribution by Polinsky and Che demonstrates that a properly employed decoupling regime can preserve the incentive for injurers (defendants) to maintain care while reducing the legal costs of lawsuits, and hence improve efficiency.

A key observation of Polinsky and Che's argument is that the injurer's optimal level of care is an increasing function of both the award to the plaintiff (W_p) and the payment by the defendant (W_d). This is the case because both W_p and W_d contribute to the injurer's expected cost of an accident, and therefore increase the incentive to exercise care. As such, starting from any coupled damages, one can raise W_d and reduce W_p at the same time, rendering the effect of holding the injurer's level of care unchanged while reducing the plaintiff's incentive to pursue the legal process and thereby reducing the litigation costs on both sides.

The purpose of this paper is to investigate the optimal decoupling design by taking into account the effects of pretrial negotiation. As pointed out by

[1] In practice the difference is collected by the government.

[2] See Sharkey (2003) pp. 375-380 for an extensive review on legislative rationale for various *split-recovery* statutes introduced in mid-1980s. These statutes represent a special form of decoupling, as they require the plaintiffs to split punitive damages with the government funds.

Nalebuff (1987), a key feature of the suit-settlement process is the information asymmetry between the litigating parties. The lack of common knowledge regarding a dispute's prospects in court often causes the failure of pretrial negotiation, and leads to jury trials that involve higher costs.[3] In other words, asymmetric information plays a crucial role in the litigating parties' strategic interaction in both the settlement-phase and the trial-phase.

The credibility of the plaintiff's threat of litigation is a central issue when considering asymmetric information. Nalebuff (1987) showed that a weaker plaintiff might actually demand a higher settlement in order to limit the bad news conveyed by the defendant's rejection of an offer. When applying this result to Polinsky and Che's model, we find a contradictory implication. In the context of decoupled liability, a plaintiff is considered weak if the award in court is set at a low level. In that event, if the plaintiff is further undermined by an even lower award, her response is likely to act more aggressively due to the credibility constraint. In other words, reducing the award may lead to a higher probability of litigation and a higher level of care from the injurer. Thus, the original argument of Polinsky and Che that an increase in the penalty matched with a decrease in the award improves efficiency is only valid when the plaintiff is not bound by the credibility concern. Accordingly, the proposed scheme that makes the defendant pay as much as he can afford is no longer optimal.

In light of this problem, our model provides a complete solution to the post-accident suit-settlement negotiation and reaches an efficient decoupling design without encouraging frivolous lawsuits or diminishing the exercise of care. That is, in our optimal system, the payment by the defendant is generally greater than the award to the plaintiff. This conclusion is in

[3]Without asymmetric information, as Rubinstein (1985) showed, all bargaining is expected to be settled immediately, and there should be no trials at all. This, of course, is inconsistent with the emphasis of trial costs in the decoupling literature.

sharp contrast with the literature such as with Polinsky and Che (1991) and Choi and Sanchirico (2004), in which the relative magnitude of penalty and award is ambiguous. We further show that the defendant's penalty can be approximated by a multiple of the harm caused. This result complements the "multiplier" theory proposed by Polinsky and Shavell (1998).[4] The amount received by the plaintiff, however, is strictly bounded and does not grow with the harm suffered. Such a decoupling scheme deters frivolous lawsuits without reducing the injurers' incentives to exercise care.

We also show that one of the goals in designing a decoupling system is to balance the litigating parties' bargaining positions in pretrial negotiation. That is, for a weak plaintiff who incurs high cost in trial, she needs to be motivated by a higher award. The extra incentive comes at the price of more lawsuits, but the injurer will be more careful. On the other hand, in the case where the plaintiff is strong and litigious, it is important to discourage her with a lower award in court, even though it comes at the cost that the injurer will be less careful.

There have been some efforts in the literature attempting to modify the decoupling analysis. Kahan and Tuckman (1995) argue that both the defendant and the plaintiff devote effort into the suit-settlement process. A decoupling causes changes in the optimal effort of both parties, which in turn yields some ambiguity in the results. Following the same approach, but fully taking into account the effects on litigation effort, Choi and Sanchirico (2004) showed that raising damages and lowering the award might not improve efficiency. They also show that, when the harm is large, the optimal

[4]These authors suggest a multiplier approach to calculate the amount of punitive damages: "the proper level of total damages ... is the harm caused multiplied by the reciprocal of the probability of being found liable." See Polinsky and Shavell (1998), p. 874. As we will show later, the multiplier in our model is higher than what they suggest due to the fact that the opportunity of settlement reduces the defendant's expected liability.

award is greater than the optimal damages, which is opposite to our conclusion. Lewis and Sappington (1999) studied another dimension of decoupling and showed that it is sometimes desirable to design a liability scheme where the defendant faces no penalty at all if the harm is small, and a very large penalty when the harm is sufficiently large. They showed that under some conditions a decoupling like this might be more efficient than a usual coupled one. Daughety and Reinganum (2003) analyzed a special version of decoupling that allows the state to share the punitive damages award. They considered a settlement bargaining model with asymmetric information, and showed that the split-award statutes leads to more frequent settlements at lower amounts. None of these articles fully characterizes the optimal decoupling system under the asymmetric information framework.

The remainder of this paper is arranged as follows. The next section introduces the basic model on which our analysis will be based. Section 3 characterizes the complete solution to our model, and shows the conditions that can validate the conventional results. In Section 4, we explicitly derive the optimal decoupling scheme and compare our results with the literature. Section 5 considers variations of our basic model and shows that the main conclusion is robust with respect to these extensions. The final section concludes.

2 The Model

Consider a model of accident and litigation similar to the one in Polinsky and Che (1991), except the information asymmetry we shall introduce later. The potential injurer and victim are both risk neutral. The injurer chooses a level of care that affects the probability of an accident. If an accident occurs, a victim is harmed, and the size of the damage is common for all victims.

When a lawsuit is filed after the accident, the probability that the victim

will prevail is q, depending on how the evidence is preserved and presented. If the victim prevails, the defendant makes a payment to the authority, and an award is given to the plaintiff. The following notations are adapted from Polinsky and Che (1991) and Nalebuff (1987).

$$\begin{aligned} c &= \text{potential injurer's level of care,} \\ p(c) &= \text{probability of an accident } (p' < 0; p'' > 0), \\ \ell &= \text{loss if an accident occurs,} \\ W_p &= \text{award to the plaintiff,} \\ W_d &= \text{payment by the defendant.} \end{aligned}$$

The fact that W_p and W_d can be different characterizes the system of decoupled liability.

Before a contingent trial, the plaintiff and the defendant may negotiate to see if a settlement can be reached. We assume that the settlement costs for the plaintiff and the defendant are negligible, and that the costs of trial for both sides are respectively

$$\begin{aligned} C_p &= \text{potential victim's (plaintiff's) trial cost,} \\ C_d &= \text{potential injurer's (defendant's) trial cost.} \end{aligned}$$

The innovation of this paper lies in the introduction of asymmetric information between the litigating parties as follows. Before the accident, the parameter of liability, q, is supposed to be drawn from a distribution $F(q)$, which is common knowledge. When an accident occurs, the defendant learns the true q by inspecting the evidence left behind, whereas the plaintiff still knows only the distribution of q. The settlement/litigation game after an accident evolves like the one in Nalebuff (1987). Specifically, the plaintiff will first make a take-it-or-leave-it settlement offer, S. If the defendant accepts S, he pays S and the plaintiff receives S.[5] If the defendant turns down S, the

[5]To simplify our analysis, we assume there is no decoupling in settlement.

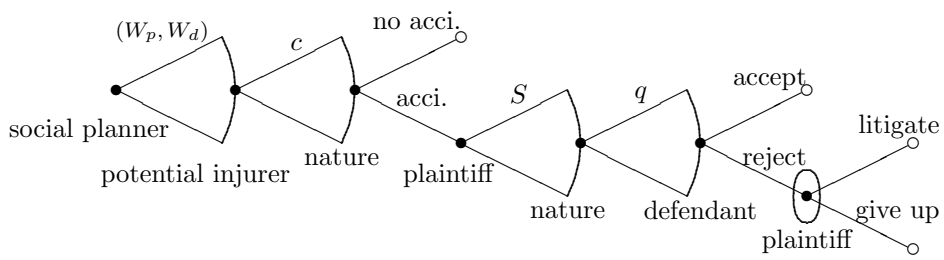

Figure 1: The game tree

plaintiff must decide whether to bring the case to court. The trial in court can reveal the true q, while both parties have to bear litigation costs. Given a decoupled liability regime, the payoffs following a trial would be $qW_p - C_p$ for the plaintiff and $-qW_d - C_d$ for the defendant. The game tree in Figure 1 illustrates how the game proceeds.

In the subgame following an accident, the plaintiff's strategy can be summarized by $(S, \alpha(S))$, where S is the settlement demand and $\alpha(S)$ is the conditional probability of litigation if S is rejected. In response to $(S, \alpha(S))$, the defendant shall adopt a "cut-off" strategy as shown in Nalebuff (1987): if the realized q is lower than a cut-off point $q(S)$, the defendant has a relatively good case so that he should reject S; if q is higher than $q(S)$, the defendant is better off settling.

Anticipating the outcome in the aforementioned subgame, the potential injurer selects a care level c that minimizes his aggregate expected costs. The injurer's costs include the cost to take precaution and the expected costs of accidents. Without loss of generality, we assume that the cost to implement c is simply c. When an accident occurs, the injurer expects to settle the case out of court with probability $1 - F(q(S))$, while he will be sued with probability $\alpha(S)F(q(S))$. Thus, the injurer's problem is to choose c so as to

minimize

$$c + p(c) \cdot \left([1 - F(q(S))]S + \alpha(S)F(q(S)) \Big[W_d \int_0^{q(S)} \frac{x dF(x)}{F(q(S))} + C_d \Big] \right). \quad (1)$$

Let c^* denote the solution that minimizes (1). Since the settlement offer in equilibrium (S^*) depends on the decoupling rates (W_p, W_d), the optimal care level c^* is also a function of (W_p, W_d). The social problem is thus to find the optimal decoupling rates that minimize the sum of the injurer's cost of care, the victim's expected harm, and both parties' expected trial costs:

$$\min_{W_p, W_d} c^* + p(c^*) \cdot (\ell + \alpha(S^*)F(q(S^*))(C_p + C_d)). \quad (2)$$

The following section derives the complete solution to the settlement game. Readers who are not interested in the bargaining analysis can move to Proposition 3 directly.

3 The Settlement Subgame

A settlement subgame is one that follows an accident in Figure 1. In the subgame, both parties are aware of the decoupled liability (W_p, W_d) as well as the care level c. In this section, we will solve for the settlement subgame equilibrium for every possible (W_p, W_d).[6] To avoid the algebraic complication of carrying higher-order differentiations of $F(\cdot)$ in our later analysis, we shall assume that the prior of q is uniformly distributed in $[0, b]$ with $b \leq 1$, so that $F(q) = q/b$.

Recall that $q(S)$ represents the defendant's cut-off strategy: given an offer S, the defendant refuses to settle if and only if $q \leq q(S)$. The following definition provides an important benchmark for $q(S)$ in characterizing the equilibrium.

[6] Since c does not affect the settlement/litigation payoffs, the subgame equilibrium is solely determined by the decoupled liability.

Definition 1. *Let q' be the critical value such that the plaintiff is indifferent between litigating and not litigating when $q(S) = q'$; q' solves the following equation.*

$$-C_p + W_p \int_0^{q'} \frac{xdF(x)}{F(q')} = 0.$$

With $F(q) = q/b$, q' is well-defined and equal to $2C_p/W_p$ if and only if $2C_p/W_p \leq b$. In the terminology of Nalebuff (1987), the plaintiff's case *has merit* if this inequality holds. When the plaintiff's case has no merit, the equilibrium in the settlement subgame is trivial, as suggested in the following proposition. It shows that the plaintiff will never sue if her reward from winning the case in court is too low. Further, the plaintiff cannot extract any payment from settlement since her threat to litigate is not credible.

Proposition 1. *Suppose $W_p < 2C_p/b$. Then in any subgame perfect equilibrium, the defendant never accepts settlement offers ($q(S) = b, \forall S > 0$), while the plaintiff never goes to court ($\alpha(S) = 0, \forall S > 0$).*

Now we consider the scenario when the case has merit. We start with the subgame in the last stage, and solve for the equilibrium strategies backwards. Suppose a settlement offer S is rejected by the defendant, whose cut-off strategy is $q(S)$. The plaintiff must determine whether to bring the case to court. The following decision rules characterize the plaintiff's best response to $q(S)$: (i) if $q(S) < q'$, the plaintiff's expected payoff of litigation is negative, and thus $\alpha(S) = 0$; (ii) if $q(S) > q'$, the expected payoff of litigation is positive, and thus $\alpha(S) = 1$; (iii) if $q(S) = q'$, the plaintiff is indifferent whether to litigate so that $\alpha(S) \in [0, 1]$.

Next, consider the subgame where the defendant is presented with a settlement offer. Suppose the plaintiff adopts the strategy $(S, \alpha(S))$. The best

response by the defendant is to accept S if and only if $q > q(S)$,[7] where

$$q(S) = \frac{\frac{S}{\alpha(S)} - C_d}{W_d}. \tag{3}$$

Note that $q > q(S)$ implies $S < \alpha(S)(qW_d + C_d)$. Therefore, the defendant is indeed better off accepting S if his realized q is greater than the cut-off value $q(S)$. Proposition 2 summarizes the equilibrium strategies in the subgame after the plaintiff has made a settlement offer S.

Proposition 2 (Nalebuff (1987)). *Assuming $W_p > 2C_p/b$, for any settlement offer $S > C_d$, the continuation subgame has a unique Nash equilibrium as follows.*

$$(q(S), \alpha(S)) = \begin{cases} \left(\frac{S-C_d}{W_d}, 1\right) & \text{if } S \geq q'W_d + C_d, \\ \left(q', \frac{S}{q'W_d + C_d}\right) & \text{if } S < q'W_d + C_d. \end{cases} \tag{4}$$

Denote the threshold in (4) by $S' \equiv q'W_d + C_d$. The proposition states that the plaintiff will go to court with probability one if the settlement offer is high enough ($S \geq S'$). In other words, a high settlement demand enables the plaintiff to "limit the bad news": the corresponding $q(S)$ from (3) will be high so that she still has a good case in court when S is rejected. Meanwhile, when the plaintiff proposes an offer below S', her threat of going to court is no longer credible.[8] Instead, the plaintiff has to reduce $\alpha(S)$ as well to maintain $q(S) = q'$ for $S < S'$. The limitation placed by the credibility consideration proves to be crucial in the settlement subgame, as shown in Nalebuff (1987).

[7] The defendant with $q = q(S)$ is indifferent between accepting and rejecting S. As long as the distribution of q is non-atomic, the strategy for the type $q(S)$ does not affect our analysis.

[8] Otherwise, $\alpha(S) = 1$ and $S < S'$ imply $q(S) < q'$ according to (3), which in turn implies $\alpha(S) = 0$, a contradiction.

When the settlement offer is even lower ($S \leq C_d$), there exist multiple equilibria in the continuation subgame. Besides $(q', \frac{S}{q'W_d+C_d})$ as prescribed in Proposition 2, $(q(S), \alpha(S)) = (0, 1)$ also constitutes an equilibrium. In the latter equilibrium, the defendant always agrees to settle because S is very low. If the defendant were to reject S, he believes that the plaintiff will sue with probability one, which is a credible threat provided $W_p > 2C_p/b$. For the convenience of exposition, we select $(q(S), \alpha(S)) = (0, 1)$ as the equilibrium for $S \leq C_d$. The selection is irrelevant because $S \leq C_d$ will never emerge in the optimal decoupling, assuming that ℓ is large enough.

Foreseeing the equilibrium response $(q(S), \alpha(S))$ that follows a settlement offer S, the plaintiff can derive her expected payoff $V(S)$ of proposing S. The following formula defines $V(S)$, which is comparable to the defendant's expected cost for an accident (cf. (1)), except that both parties bear their own litigation costs, and the liabilities are decoupled.

$$V(S) \equiv (1 - F(q(S)))S + \alpha(S)F(q(S)) \left(-C_p + W_p \int_0^{q(S)} \frac{x dF(x)}{F(q(S))} \right). \quad (5)$$

Assuming the case has merit and q is uniformly distributed, $V(S)$ is given as follows.

$$V(S) = \begin{cases} S & \text{if } S \leq C_d, \\ \left(1 - \frac{q'}{b}\right) S & \text{if } C_d < S \leq S', \\ \left(1 - \frac{S-C_d}{bW_d}\right) S + \frac{S-C_d}{bW_d} \left(-C_p + \frac{S-C_d}{bW_d} \frac{bW_p}{2} \right) & \text{if } S' < S < \bar{S}, \\ -C_p + \frac{bW_p}{2} & \text{if } S \geq \bar{S}, \end{cases} \quad (6)$$

where $\bar{S} \equiv bW_d + C_d$. For $S \leq C_d$, one obtains $V(S) = S$ by substituting the selected equilibrium, $(q(S), \alpha(S)) = (0, 1)$, into (5). For the other three cases, the equilibrium is unique from Proposition 2, and we simply apply (4) to derive $V(S)$. For $C_d < S \leq S'$, $q(S) = q'$ and thus the net payoff of a lawsuit is zero by definition of q'. $V(S)$ in this case reflects only the

settlement income. In contrast, for $S \geq \bar{S}$ in the last case, $q(S) \geq b$, which means that no defendant will accept such a high settlement offer. $V(S)$ here reflects only the net payoff from litigation. Finally, for $S' < S < \bar{S}$, $q(S)$ lies between q' and b. The plaintiff derives her expected payoff from both settlement and litigation.

Note that the value function $V(S)$ is continuous except when S is equal to C_d. In addition to the jump at C_d, $V(S)$ also has two kinks at $S = S'$ and $\bar{S}$. The first kink is due to the credibility constraint: when S drops below S', the threat that $\alpha(S) = 1$ is no longer credible. As for the second kink, $V(S)$ is constant for $S \geq \bar{S}$ because no defendant will agree to a settlement demand higher than $\bar{S}$.

Provided (6), the plaintiff's optimization problem in the settlement subgame is to select an optimal settlement demand S^* to maximize $V(S)$. Let $\widehat{V}(S)$ denote the unconstrained value function (i.e., the segment of $V(S)$ for $S \in (S', \bar{S})$).

$$\widehat{V}(S) \equiv \left(1 - \frac{S - C_d}{bW_d}\right) S + \frac{S - C_d}{bW_d}\left(-C_p + \frac{S - C_d}{bW_d}\frac{bW_p}{2}\right). \tag{7}$$

The following definition will be useful in characterizing S^*.

Definition 2. *$\widehat{S}$ solves the equation $d\widehat{V}(S)/dS = 0$ while $\widehat{q} \equiv \frac{\widehat{S} - C_d}{W_d}$.*

In other words, $\widehat{S}$ maximizes $V(S)$ if the credibility constraint is not binding *and* the second order condition holds. For uniformly distributed q, one obtains $\widehat{q} = \frac{bW_d - C_p - C_d}{2W_d - W_p}$.

From the above discussion, we know that the optimal S^* must be either $\widehat{S}$ or one of the corner solutions (C_d, S', or, $\bar{S}$). Which of these solutions maximizes $V(S)$ depends on the decoupled liability (W_p, W_d). Proposition 3 shows that one can partition the set, $\omega \equiv \{(W_p, W_d) : W_p > 2C_p/b,\ W_d > 0\}$, into four subsets such that each subset associates with a certain solution for S^*.[9]

[9] The specific definitions of $\omega's$ are delegated to (A1) in Appendix.

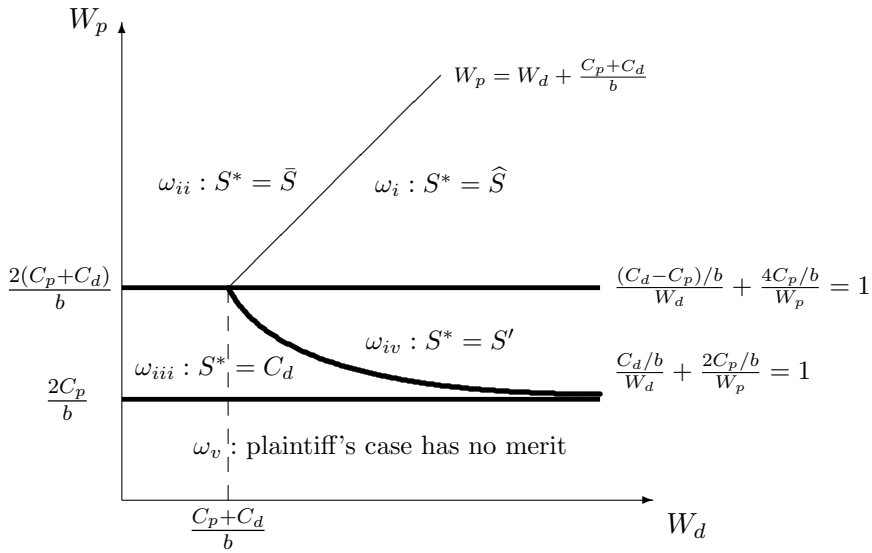

Figure 2: Optimal settlement demand S^*

Proposition 3. *Assuming $W_p > 2C_p/b$, the optimal settlement demand S^* as a function of (W_p, W_d) is given by $\widehat{S}$, $\bar{S}$, C_d, or S' if $(W_p, W_d) \in \omega_i$, ω_{ii}, ω_{iii}, or ω_{iv}, respectively, where the regions $\omega_i \cdots \omega_{iv}$ are as depicted in Figure 2.*

Proof. See Appendix. ■

Note that the curve representing $\frac{(C_d-C_p)/b}{W_d} + \frac{4C_p/b}{W_p} = 1$ is a flat line as shown in the diagram if $C_d = C_p$. It will be increasing (decreasing) and concave (convex) if $C_d < C_p$ ($C_d > C_p$).[10]

[10] On the boundaries between $\omega's$, the plaintiff is indifferent between various choices of S^*. For instance, when $W_p = \frac{2(C_p+C_d)}{b}$ and $W_d \leq \frac{C_p+C_d}{b}$, the plaintiff is indifferent between proposing C_d or $\bar{S}$. One can verify that $\widehat{S} = \bar{S}$ for $(W_p, W_d) \in \omega_i \cap \omega_{ii}$ and $\widehat{S} = S'$ for $(W_p, W_d) \in \omega_i \cap \omega_{iv}$. Therefore, $S^*(W_p, W_d)$ as stated in Proposition 3 coincides along these two boundaries. However, $\bar{S}$ can never be equal to C_d (b cannot be zero), and thus S^* is discontinuous on the border between ω_{ii} and ω_{iii}. Likewise, S' can never be equal to C_d (q' cannot be zero), and thus S^* is also discontinuous on the border between ω_{iii} and

Propositions 1 and 3 characterize the settlement subgame equilibria for $W_p < 2C_p/b$ and $W_p > 2C_p/b$, respectively. With similar arguments, one can show that all of the subgame equilibria in Propositions 1 and 2 remain to be equilibrium in the scenario with $W_p = 2C_p/b$. As we will show later, the equilibria that associate with $W_p = 2C_p/b$ will never emerge in the optimal decoupling, assuming a sufficiently large ℓ. Therefore, how we select the equilibrium is irrelevant to our analysis.

4 The Optimal Decoupling

The previous section characterizes the optimal settlement demand S^* for any given decoupling rates, (W_p, W_d). Accordingly, the injurer derives his expected payment for an accident (EPA) as a function of (W_p, W_d).

$$EPA \equiv \left(1 - \frac{q(S^*)}{b}\right) S^* + \frac{\alpha(S^*)q(S^*)}{b}\left(\frac{q(S^*)}{2}W_d + C_d\right).$$

Before any accident occurs, the potential injurer chooses a care level c to minimize his aggregate expected costs, $c + p(c) \cdot EPA$. In view of (1), one obtains the optimal c by solving the first order condition as follows.

$$1 + p'(c) \cdot EPA = 0. \tag{8}$$

The regularity assumptions of $p'(c) < 0$ and $p''(c) > 0$ assure that a solution to (8) indeed minimizes the injurer's aggregate expected costs. In addition, they imply that an increase in EPA induces a higher care level by the potential injurer.[11] Let $c^*(W_p, W_d)$ denote the solution to (8). The social problem

ω_{iv}. For convenience of exposition, we define ω_{iii} such that it is disjoint with the other three subsets. Assuming a sufficiently large ℓ, the way we select S^* along these boundaries does not affect our analysis because they never emerge as the optimal decoupling.

[11] That is, $dc/dEPA = p'(c)^2/p''(c) > 0$.

is to select (W_p, W_d) that minimizes the social cost as described in (2):

$$\min_{W_p, W_d} c^* + p(c^*) \cdot \left(\ell + \frac{\alpha(S^*)q(S^*)}{b}(C_p + C_d) \right). \tag{9}$$

As we have shown in the last section, $\omega's$ divide the liability space into various regions, each of which corresponds with a particular solution of S^*. By analyzing the optimization program in (9) separately for each region, one concludes that a necessary condition for a liability system to achieve efficiency is $(W_p, W_d) \in \omega_i \cap \omega_{iv}$. We briefly discuss the properties of these regions. The details can be found in Appendix.

- In ω_i, the credibility constraint is not binding, and thus the interior solution is feasible so that $S^* = \widehat{S}$. Proposition 4 below shows that Polinsky and Che (1991)'s argument still applies in this region. It follows that any interior point in ω_i cannot be efficient (Lemma 2).

- In ω_{ii}, the award W_p is very high comparing to the penalty W_d. Consequently, both parties prefer to resolve the case in court instead of settlement. Lemma 1 shows that to decouple liabilities in this way can never be optimal.

- In ω_{iii}, the award is so low that the plaintiff would rather settle the case out of court with $S^* = C_d$. As we argue in Lemma 4, when the harm caused is sufficiently large, this type of decoupled liability cannot be efficient since it does not provide enough incentive for the injurer to take adequate precautions.

- In ω_{iv}, the credibility constraint is binding, and $S^* = S'$. Proposition 4 shows that one can adapt Polinsky and Che (1991)'s argument by raising both W_d and W_p to enhance efficiency. Therefore, any interior point in ω_{iv} cannot be efficient (Lemma 2).

- Finally, the plaintiff's case is meritless in ω_v. One can verify that making the case meritless cannot be efficient, provided $\ell > C_d$ (Lemma 3). The reason is similar to that for the scenario ω_{iii}: the injurer has no incentive to be careful when causing an accident has no consequence.

Before we proceeds with solving the optimization program in (9), we shall first explain why the argument and solution proposed in Polinsky and Che (1991) are different when there is asymmetric information concerning q. Polinsky and Che (1991) argue that the social planner can improve efficiency of the coupled liability system by raising W_d and reducing W_p at the same time. On the one hand, raising W_d increases the injurer's expected costs when an accident occurs, and thus encourages him to be more careful. On the other hand, reducing W_p decreases the victim's return from litigation and hence her incentive to sue, which in turn induces less care from the injurer. If one adjusts W_d and W_p in such a way that the injurer's level of care remains the same, the cost of care is not affected but the litigation costs can be saved, which results in lower social cost.

The following proposition shows that the scheme proposed by Polinsky and Che improves efficiency only when the credibility constraint is not binding. If the credibility constraint is binding, one could reduce the social cost with an alternative scheme that raises both W_d and W_d.

Proposition 4. *For any (W_p, W_d) in $\omega_i \setminus \omega_{iv}$, there exist $\triangle_p, \triangle_d > 0$ such that the new decoupled liability $(W_p - \triangle_p, W_d + \triangle_d)$ is more efficient with the same level of care but fewer lawsuits. In contrast, the efficiency-improving scheme takes a different form when starting from ω_{iv}: $\forall\ (W_p, W_d) \in \omega_{iv} \setminus \omega_i$, $\exists \triangle_p, \triangle_d > 0$ such that $(W_p + \triangle_p, W_d + \triangle_d)$ implements the same level of care with fewer lawsuits.*

Proof. See Appendix. ■

Proposition 4 indicates that raising W_d and lowering W_p indeed reduce the social cost *if* we start from $(W_p, W_d) \in \omega_i$. However, when W_d and W_p keep moving in the prescribed directions, they eventually reach the quadrant ω_{iv} where $S^* = S'$ (see Figure 2). Nonetheless, $\frac{\partial C^*}{\partial W_p}$ and $\frac{\partial q(S^*)}{\partial W_p}$ are both negative in this subset, and thus further reducing W_p does not render the desired effects suggested by Polinsky and Che (1991).

The next proposition shows that an alternative system is more efficient as long as the accident loss ℓ is sufficiently large.

Assumption 1. $(1 - p(c))$ *is a probability distribution function. That is,* $p(0) = 1$ *and* $\lim_{c\to\infty} p(c) = 0$.

Assumption 2. *The ratio,* $\frac{-p'(c)}{p(c)}$, *is an increasing function of* c.

$\frac{-p'(c)}{p(c)}$ can be interpreted as the hazard rate when Assumption 1 holds. Note that the corresponding density function is $-p'(c)$ so that the usual definition of hazard rate applies.

Proposition 5. *Suppose Assumptions 1 and 2 hold. There exists* $\underline{\ell}$ *such that for* $\ell > \underline{\ell}$, *the social planner's optimization problem in* (9) *reduces to*

$$\begin{aligned} \min_{W_d, q^*, c^*} \; & c^* + p(c^*) \cdot \left(\ell + \frac{q^*}{b}(C_p + C_d) \right) \\ s.t. \quad & q^* = \frac{b}{2} - \frac{C_d - C_p}{2W_d}, \\ and \quad & 1 + p'(c^*) \cdot \left(q^* \left(1 - \frac{q^*}{2b} \right) W_d + C_d \right) = 0. \end{aligned} \tag{10}$$

Proof. See Appendix. ■

The first constraint that determines q^* is equivalent to the condition that $(W_p, W_d) \in \omega_i \cap \omega_{iv}$.[12] Recall that the plaintiff's credibility constraint is

[12] By substituting $q^* = \frac{2C_p}{W_p}$ into $\frac{(C_d - C_p)/b}{W_d} + \frac{4C_p/b}{W_p} = 1$ that defines $\omega_i \cap \omega_{iv}$, one obtains $q^* = \frac{b}{2} - \frac{C_d - C_p}{2W_d}$.

binding in ω_{iv}, and non-binding in ω_i. Proposition 5 shows that for any decoupled liability to be efficient, the credibility constraint must be *marginally* binding, provided ℓ large enough.

The solution to (10) depends on the functional form of $p(c)$ as well as other parameters like ℓ, C_p, or C_d. In general, there is no closed-form solution to the decoupling problem. Nonetheless, the optimal decoupled liability takes a very simple form when $C_p = C_d$.

Example 1. *Suppose $C_p = C_d$. The optimal decoupling that solves* (10) *is*

$$(W_p, W_d) = \left(\frac{4C_d}{b}, \frac{8\ell}{3b}\right).$$

By further assuming that $p(c) = \exp(-c)$, one obtains $\underline{\ell} \approx 2.146 \cdot C_d$. In other words, the decoupling system that solves (10) *is the most efficient one if and only if $\ell > 2.146 \cdot C_d$.*

Proof. See Appendix. ■

In the example, we find that the optimal award to the plaintiff is independent of the loss suffered. In contrast, the optimal payment by the defendant is greater than and proportional to ℓ. The following corollaries generalize these properties.

Corollary 6. *In the optimal decoupling, the plaintiff's award is confined to an interval:*

$$W_p \in \left[\frac{2C_p + 2\min(C_p, C_d)}{b}, \frac{2C_p + 2\max(C_p, C_d)}{b}\right]. \tag{11}$$

The upper and lower bounds of the interval are independent of ℓ.

Proof. The corollary follows from the observation that the path of $\omega_i \cap \omega_{iv}$ is bounded between $W_p = \frac{2(C_p+C_d)}{b}$ and the asymptotic line, $W_p = \frac{4C_p}{b}$. ■

An important policy implication from Corollary 6 is that the court should abstain from awarding the plaintiff excessive damages. Even though a higher W_p may impel the defendant to be more careful, the benefit is outweighed by the increase of wasteful lawsuits.

Corollary 7. *Suppose Assumption 2 holds. There exists $\underline{\ell}'$ such that for $\ell > \underline{\ell}'$, the optimal W_d has a lower bound. In particular, $W_d > \frac{2\ell}{b}$ if $C_d < C_p$, while $W_d > \frac{8(\ell - C_d)}{3b}$ if $C_d > C_p$.*

Proof. See Appendix. ■

Combining the above characterizations of the optimal decoupling rates, one concludes that, when the harm caused is sufficiently large,[13] the optimal penalty is greater than the optimal award since the latter is bounded while the former grows with the harm caused.

Corollary 7 implies that the court should impose punitive damages in addition to compensatory damages ($W_d > \ell$). Moreover, the magnitude of the total damages should be aligned with ℓ to achieve the optimal deterrence. In fact, the following equation provides a simple formula to determine the optimal penalty in the limiting case.

$$\ell + \frac{q^*}{b}(C_p + C_d) = q^* \left(1 - \frac{q^*}{2b}\right) W_d + C_d, \tag{12}$$

where $q^* = \frac{b}{2}$.[14] This result complements the multiplier theory proposed by Polinsky and Shavell (1998).[15] Assuming uniform distribution, the multiplier

[13] In Example 1, for instance, the inequality $\frac{4C_d}{b} < \frac{8\ell}{3b}$ holds whenever $\ell > \underline{\ell}$. Thus, the requirement that *the harm caused be sufficiently large* is not too stringent.

[14] Recall that the injurer solves $1 + p'(c^*) \cdot EPA = 0$ to obtain c^*, where EPA is given by the right hand side in (12). Moreover, as $\ell \to \infty$, q^* converges to $\frac{b}{2}$ and $\frac{dq^*}{dW_d}$ converges to zero. Thus, the first order condition to (10) reduces to $1 + p'(c^*) \cdot (\ell + \frac{q^*}{b}(C_p + C_d)) = 0$ (cf. (A3)). One obtains (12) by comparing these two conditions.

[15] See footnote 4.

granted by their formula is 2, which is "the reciprocal of the probability of being found liable." In contrast, the multiplier obtained from (12) is $\frac{8}{3}$. The discrepancy is due to the fact that the opportunity of settlement reduces the defendant's expected liability.[16]

The path of $\omega_i \cap \omega_{iv}$ in Figure 2 depicts the necessary condition that regulates the optimal decoupling rates. For $C_p > C_d$, one observes that an increase in W_d leads to a higher W_p and fewer lawsuits (i.e., $\omega_i \cap \omega_{iv}$ has a positive slope). The intuition is that, with $C_p > C_d$, it is relatively more difficult for the plaintiff to establish credibility to sue. The concern of credibility forces the plaintiff to litigate excessively. Raising W_d improves the plaintiff's bargaining position and alleviates her credibility problem. As a result, the plaintiff does not need to litigate as much as before. Conversely, for $C_p < C_d$, a higher W_d associates with a lower W_p and more lawsuits ($\omega_i \cap \omega_{iv}$ has a negative slope). The reason is that the plaintiff here is too confident about her case in court due to her relatively low trial cost. Raising W_d will only encourage her to act even more aggressively and hurt the chance of settlement.

The previous discussion addresses the marginal effects of W_d on settlement rates. Essentially, it depends on whether the plaintiff is "confident" about her case: she is confident in court when $C_p < C_d$. The same condition plays a significant role in characterizing the global properties of the optimal decoupling system, as we will show in the next corollary.

Corollary 8. *Suppose $\ell > \underline{\ell}$. The inequality $C_p < C_d$ implies that the settlement rate in equilibrium is higher than $\frac{1}{2}$, and that the care level is lower than the socially optimal level.*

Proof. See Appendix. ■

[16] Without possibility of settlement, the expected cost of an accident (EPA) for the injurer is equal to $\frac{b}{2}W_d + C_d$, which is greater than the right hand side of (12).

The first assertion indicates that a relatively stronger plaintiff (in terms of trial costs) acts less aggressively and settle the case more often. The second assertion characterizes the response from the defendant: when defending against a less aggressive plaintiff, the injurer tends not to be motivated to take adequate precautions.

5 Extension and Discussion

5.1 Comparative statics

This section studies the properties of the optimal decoupling system, assuming the accident loss ℓ is sufficiently large so that Proposition 5 applies. We will show that, with a small increase of ℓ, the social planner should impose a higher penalty payable by the defendant, that the injurer will be more careful, and that the victim will make a higher settlement demand when an accident occurs. Nonetheless, the impact on the amount awarded to the victim and the chance of settlement depends on which party incurs a higher litigation cost.

Proposition 9. *Assuming $\ell > \underline{\ell}$, a small increase in ℓ leads to an increase in the defendant's liability W_d, a higher care level, and a higher settlement demand. It also leads to an increase in the plaintiff's award W_p, and a higher probability of settlement if and only if $C_d < C_p$.*

Proof. See Appendix. ■

The social costs comprise cost of care and expected costs of accidents. When the damage caused by an accident is higher, the second type of cost outweighs the first. From the society's point of view, the chance of accident needs to be lower to return to balance. In order to make sure that the

potential injurer exercises extra care that reduces $p(c)$, the social planner shall raise W_d, which leads to a higher EPA and a higher level of care.

The impact of a higher ℓ on W_p and q^* is not as straightforward. Even though an increase in q^* assures a higher level of care in view of (A2), it also results in more litigation and thus higher trial costs. The tradeoff between the chance and the costs of an accident is balanced according to (10) so that the plaintiff's credibility constraint is marginally binding.

It is important to note that changes in ℓ do not alter the diagram in Figure 2. Therefore, the new optimal decoupling rates due to a higher ℓ must stay at the same trajectory of $\omega_i \cap \omega_{iv}$, and consequently, q^* is determined by the same equation as (10). In sum, a small increase in ℓ leads to a higher W_d, which determines q^* through (10).

In conclusion, when the damage from an accident is more severe, the social planner's goal is to induce a higher level of care from the injurer. In addition to raising W_d to let the defendant internalize the damage, the social planner also needs to encourage a weak plaintiff (with $C_p > C_d$) to settle by providing a higher award in court, and urge a strong plaintiff to litigate by reducing W_p.

5.2 Budget constraint

In their model with complete information, Polinsky and Che (1991) have argued that the defendant should make the maximum possible payment in the optimal decoupling. By incorporating their assumption of budget constraint, we show that their conclusion can emerge in equilibrium as a special case of our model.

Suppose the defendant's payment cannot exceed an upper bound, m. The social problem in this scenario is to find a solution to (10), with the additional constraint that $W_d \leq m$. Obviously, if the budget constraint is not binding

(i.e., the optimal payment that solves (10) is below the upper bound, m), the optimal decoupling will not be affected by the budget constraint. If, however, the budget constraint is binding, the optimal decoupling under the constraint is exactly the system that has been proposed by Polinsky and Che.

Proposition 10. *If the budget constraint is binding so that the unconstrained optimal payment is greater than m, the optimal decoupling with budget constraint should make the defendant's payment as high as possible, i.e., the optimal payment is equal to m.*

The argument follows from Proposition 4. A decoupling system in $\omega_i \setminus \omega_{iv}$ can always be improved upon by raising W_d and reducing W_p, while a system in $\omega_{iv} \setminus \omega_i$ can be improved upon by raising both W_d and W_p. Without the budget constraint, the social planner can limit her search for the optimal decoupling to the boundary, $\omega_i \cap \omega_{iv}$, as we have shown in Proposition 5. In the presence of the budget constraint, the social planner must extend her search to include the vertical line, $W_d = m$, because the efficiency-improving scheme (by reducing lawsuits while maintaining the care level) may reach $W_d = m$ first before it arrives at $\omega_i \cap \omega_{iv}$. When the budget constraint is binding, the social cost is decreasing in W_d along the path of $\omega_i \cap \omega_{iv}$. In that case, the optimal W_d is equal to m.

5.3 Endogenous distribution of q

We have assumed that the distribution of q is independent of the injurer's effort to take precaution. In this section, we will show that our conclusion is robust to this assumption.

Suppose the injurer's liability q follows a uniform distribution over $[0, b(c)]$ with $b'(c) < 0, b''(c) > 0$. The upper bound $b(c)$ (and thus the distribution of q) is fixed after the injurer selects her care level. The subsequent settlement subgame proceeds exactly as discussed in Section 3.

A diagram similar to that in Figure 2 illustrates how the plaintiff selects her optimal settlement demand. As we assume an endogenous distribution of q, the subsets $\omega's$ that characterize S^* will be determined endogenously as well. For instance, the plaintiff's case has merit if $W_p > \frac{2C_p}{b(0)}$. Term b in the threshold is given by $b(0)$ because the injurer will not exercise any care at all when the victim's case has no merit.

The following numeric example is analogous to Example 1 except that the distribution of q is endogenous.

Example 2. *Suppose $C_p = C_d$, $p(c) = \exp(-c)$, and $b(c) = \exp(-c)$. The optimal decoupled liability is given as follows, provided $\ell > 2.146 \cdot C_d$.*

$$(W_p, W_d) = \left(\frac{4C_d}{b}, \frac{2\ell}{b}\right), \quad b = \frac{1}{\ell + C_d}.$$

Proof. See Appendix. ■

It is interesting to note that in equilibrium the injurer's care level as well as the social costs are exactly the same as those in Example 1. However, we have shown in the proof that under any decoupling system, the injurer is more motivated to take precautions if it reduces his share of responsibility. These contradictory facts are only superficial: note that the optimal W_d in the current example is actually lower than before. In other words, when the injurer is self-motivated, the social planner does not have to impose severe punishment in court anymore.

5.4 Negligence rule

In the discussion so far, the parameter q represents the plaintiff's chance of winning the lawsuit. Alternatively, q can be interpreted as the defendant's share of responsibility in the accident. In the latter interpretation, the court adopts a simple liability rule in which the defendant is always liable regardless

of q, and the amount of damages he pays is the predetermined W_d, multiplied by q. In practice, a certain negligence rule is often in place. In this section, we shall study the impact of negligence rules on the injurer's and the victim's behaviors, as well as the implication for the social planner's policy choice.

Consider the original model in Section 2, except that the defendant is not liable for any damages if his share of responsibility q in the accident is lower than a threshold, q_d. In other words, the defendant is deemed negligent and pays qW_d to the authority if and only if q revealed in court is higher than the due-care standard, q_d. Whatever the court judgment is, both parties pay their own trial costs.

To solve for the equilibrium of the current settlement subgame, one adapts the benchmark q' in Definition 1 as follows.

$$-C_p + W_p \int_{q_d}^{q'} \frac{xdF(x)}{F(q')} = 0.$$

The lower bound for the integration has changed from 0 to q_d since the plaintiff loses the case if $q < q_d$ due to the negligence rule. Assuming the same uniform distribution for q, one obtains $q' = \frac{C_p}{W_p} + \sqrt{\left(\frac{C_p}{W_p}\right)^2 + q_d^2}$. q' is higher than before because the plaintiff needs a better case to break even in court under the new negligence rule. To put it differently, the threshold of W_p for the case to have merit is higher than the previous threshold $2C_p/b$.

Proposition 1 still applies, with an adapted presumption for meritless cases.[17] Proposition 2 holds as well, with the new q'. Nonetheless, for a settlement offer lower than C_d, $(q(S), \alpha(S)) = (0, 1)$ can no longer constitute an equilibrium. In fact, for any $q(S) < q'$, the plaintiff will abandon the case in the last stage. The reason is that some defendants (with $q < q_d$) will never settle as they are certain that they are not at fault. Therefore, the threat to litigate is always on the equilibrium path,[18] and hence not credible if

[17] Specifically, the case does not have merit if $W_p < 2bC_p/(b^2 - q_d^2)$.

[18] In Section 3 without the negligence rule, the threat is off the path for $(q(S), \alpha(S)) =$

$q(S) < q'$. Consequently, there exists a unique equilibrium for $S \leq C_d$, which is the same as that prescribed in Proposition 2: $(q(S), \alpha(S)) = \left(q', \frac{S}{q'W_d + C_d}\right)$.

To determine the optimal settlement demand, the plaintiff maximizes her expected value $V(S)$ of proposing S:

$$V(S) \equiv (1 - F(q(S)))S + \alpha(S)F(q(S)) \left(-C_p + W_p \int_{q_d}^{q(S)} \frac{xdF(x)}{F(q(S))}\right).$$

Once again, the only change in the formulation due to the negligence rule is the lower bound of the integration.

There are three candidates for S that could maximize $V(S)$: the interior solution of $S = \widehat{S}$ and two corner solutions of $S = S'$ or $\bar{S}$.[19] Figure 3 depicts the partitions of the decoupling-rate space that characterize the equilibria in the settlement subgame. With analogous argument that leads to Proposition 5, one can show that a necessary condition for the optimal decoupling is given by $q' = \widehat{q}$, which characterizes the border between the regions of $S^* = S'$ and $S^* = \widehat{S}$ in Figure 3.

The general impact of q_d on the equilibrium is ambiguous. Nonetheless, under the same decoupling rates, we know that the injurer will be less careful for a small increase of q_d when q_d is close to zero.[20] The reason is quite obvious: the existence of a negligence rule partly relieves the injurer of his responsibility. At the margin, the negligence rule alters the bargaining positions in the settlement subgame and favors the defendant. In response, the plaintiff becomes more litigious, which leads to a higher social cost per accident. It follows that the social planner should raise W_d when implementing the negligent rule so that the injurer will be more careful in the new system.

$(0, 1)$, since every defendant accepts S with $q(S) = 0$.

[19] $S = C_d$ is not eligible as $(q(S), \alpha(S)) = (0, 1)$ no longer constitutes an equilibrium.

[20] Specifically, $\frac{\partial EPA}{\partial q_d}$ converges to $-\frac{C_d}{b}$ as q_d approaches zero, and thus $\frac{dc^*}{dq_d} < 0$ in a neighborhood of $q_d = 0$.

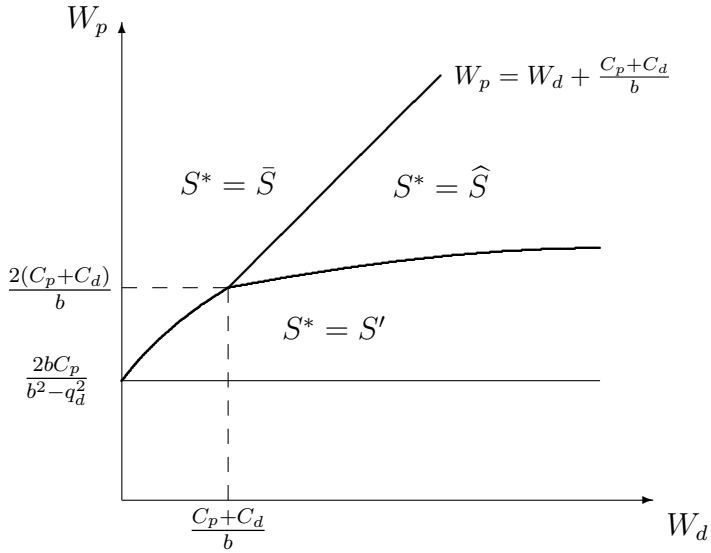

Figure 3: Optimal settlement demand with a negligence rule

6 Conclusion

In this paper, we have studied the optimal system of decoupled liability. When asymmetric information between injurer and victim is present, the concern of credibility plays a crucial role in the plaintiff's pretrial settlement strategy. In essence, if a plaintiff worries too much about her credibility in court, she acts too aggressively in the settlement phase. Accordingly, the injurer makes an excessive effort to prevent an accident from happening in order to avoid an aggressive victim. Conversely, if a plaintiff is too confident about her case, she would rather bring the case to court than settle it. In either case, there will be too much litigation, leaving the system inefficient. Thus, the first goal in designing a decoupled liability system is to balance the bargaining positions for those involved in an accident.

The second goal of a liability system is to make sure that the injurer takes *adequate* precautions. The tradeoff here is the cost of precautionary effort, and the benefit it creates by reducing the chance of future accidents. When

there is no asymmetric information, it is easy to verify that $W_d = 2(\ell - C_d)/b$ implements the first-best liability system in which there is no litigation, and the injurer selects a care level that internalizes future potential harm. The presence of asymmetric information makes wasteful lawsuits unavoidable. Otherwise, the injurer will never agree to settle. Consequently, the second-best care level is distorted upward.

The design proposed by Polinsky and Che (1991) cannot accomplish the above-mentioned goals when the plaintiff is uninformed about the strength of her case. With the penalty raised and the award reduced, the plaintiff will be undermined so that she has to establish her credibility through litigation. Meanwhile, making the penalty as high as possible forces the injurer to take excessive care that might be wasteful. Nonetheless, when the defendant in our model cannot afford the optimal penalty, he should pay as much as he can. In this scenario, our solution coincides with the optimal system suggested by Polinsky and Che. That is, our model departs from that of Polinsky and Che only when the defendant has deep pockets.

Presumably, the plaintiff can be awarded more than what the defendant pays. Nonetheless, we show that it does not happen in equilibrium. The assertion is most evident in the limiting case when the harm is sufficiently large. In that scenario, we demonstrate that the optimal penalty can be approximated by a multiple of the actual harm (the multiplier is 8/3, assuming uniform distribution). In contrast, the optimal award is always confined in a range independent of the harm suffered by the plaintiff.

As we have demonstrated, to achieve the optimal deterrence, one must balance the litigating parties' bargaining positions in pretrial negotiation. We show that a weaker plaintiff needs to be motivated through a higher award in court, which leads to more lawsuits and higher care level in equilibrium. Conversely, when the plaintiff is stronger in terms of trial costs, the award should be set lower, which results in less litigation at the cost that the injurer

now exerts less care. Similar arguments can be applied to the analysis of various liability systems. For instance, a negligence rule essentially enhances the defendant's bargaining position. When the social planner adjusts the decoupled liability accordingly, it results in a lower settlement rate, but fewer accidents.

Appendix

Proof of Proposition 3

The following expression defines the subsets that correspond with various S^* when the case has merit.

$$\begin{aligned}
\omega_i &\equiv \left\{(W_p, W_d) \in \omega : \ \frac{(C_d - C_p)/b}{W_d} + \frac{4C_p/b}{W_p} \leq 1, \ W_p \leq W_d + \frac{C_p + C_d}{b}\right\}, \\
\omega_{ii} &\equiv \left\{(W_p, W_d) \in \omega : \ W_p \geq W_d + \frac{C_p + C_d}{b}, \ W_p \geq \frac{2(C_p + C_d)}{b}\right\}, \\
\omega_{iii} &\equiv \left\{(W_p, W_d) \in \omega : \ W_p < \frac{2(C_p + C_d)}{b}, \ \frac{C_d/b}{W_d} + \frac{2C_p/b}{W_p} > 1\right\}, \\
\omega_{iv} &\equiv \left\{(W_p, W_d) \in \omega : \ \frac{C_d/b}{W_d} + \frac{2C_p/b}{W_p} \leq 1, \ \frac{(C_d - C_p)/b}{W_d} + \frac{4C_p/b}{W_p} \geq 1\right\}.
\end{aligned} \tag{A1}$$

First, note that the boundaries of $\omega's$ join at

$$(W_p, W_d) = \left(\frac{2(C_p + C_d)}{b}, \frac{C_p + C_d}{b}\right).$$

Therefore, the following equations

$$\begin{aligned}
W_p &= \frac{2(C_p + C_d)}{b}, & \frac{C_d/b}{W_d} + \frac{2C_p/b}{W_p} &= 1, \\
W_p &= W_d + \frac{C_p + C_d}{b}, & \frac{(C_d - C_p)/b}{W_d} + \frac{4C_p/b}{W_p} &= 1,
\end{aligned}$$

represent the boundaries of $\omega's$ and separate ω into four quadrants. We want to show that each quadrant associates with a particular S^*.

Consider the case with $W_p > 2W_d$. From (7), $\widehat{V}(S)$ is a quadratic function with a positive coefficient on S^2. Moreover, $\widehat{V}(S)$ attains the same value as $V(S)$ at $S = C_d$. Therefore, $V(S)$ is maximized at either $S = C_d$ or $\bar{S}$. Comparing $V(C_d)$ and $V(\bar{S})$ implies that $W_p = \frac{2(C_p+C_d)}{b}$ separates the sets of $\{S^* = \bar{S}\}$ and $\{S^* = C_d\}$.

When $W_p = 2W_d$, $\widehat{V}(S)$ is a linear function. The coefficient on S is positive if and only if $W_p > \frac{2(C_p+C_d)}{b}$, in which case $S^* = \bar{S}$. Otherwise, $\widehat{V}(S)$ is decreasing, and $S^* = C_d$.

With $W_p < 2W_d$, $\widehat{V}(S)$ is a quadratic function with a negative coefficient on S^2. Thus, $V(\bar{S})$ is maximal if and only if $\widehat{S} \geq \bar{S}$, which is equivalent to $W_p \geq W_d + \frac{C_p+C_d}{b}$. If the latter inequality does not hold, we must determine whether the interior solution $\widehat{S}$ is feasible. If the answer is yes, $S^* = \widehat{S}$; otherwise, we compare $V(S')$ against C_d. $\widehat{S}$ is feasible if and only if $\widehat{S} \geq S'$, which reduces to $\frac{(C_d-C_p)/b}{W_d} + \frac{4C_p/b}{W_p} \leq 1$. Comparing $V(S')$ with C_d is straightforward. ■

Proof of Proposition 4

Recall that the defendant's expected payment for an accident, when $(W_p, W_d) \in \omega_i \cup \omega_{iv}$, is given as follows.

$$EPA = q^* \left(1 - \frac{q^*}{2b}\right) W_d + C_d, \tag{A2}$$

where $q^* = \widehat{q}$ or q' for $(W_p, W_d) \in \omega_i$ or ω_{iv}, respectively. EPA determines the injurer's care level according to the first order condition in (8).

For $(W_p, W_d) \in \omega_i$, $q^* = \widehat{q} \equiv \frac{bW_d - C_p - C_d}{2W_d - W_p}$. In view of (A2), the set $\{(W_p, W_d) : EPA = constant)\}$ defines an *iso-cost* curve. It is tedious but straightforward to verify that the iso-cost curves in ω_i have negative slope. It follows that for any $(W_p, W_d) \in \omega_i \backslash \omega_{iv}$, there exists $\triangle_p, \triangle_d > 0$ such that the new decoupled liability $(W_p - \triangle_p, W_d + \triangle_d)$ stays in the region ω_i and belongs to the same iso-cost curve as (W_p, W_d). To show that $(W_p - \triangle_p, W_d + \triangle_d)$

implies a lower q^*, note that it corresponds with the same EPA as (W_p, W_d). Since $q^*(1 - \frac{q^*}{2b})$ is an increasing function of q^*, a higher W_d must associate with a lower q^* to keep EPA constant.

For $(W_p, W_d) \in \omega_{iv}$, $q^* = q' \equiv \frac{2C_p}{W_p}$. It is easier to see that the iso-cost curves in this region have positive slope. The rest of the argument is analogous to the previous scenario. ■

Proof of Proposition 5

The proof proceeds in four steps. Each step is summarized in a lemma. The first lemma proves that any $(W_p, W_d) \in \omega_{ii}$ cannot solve (9). The second lemma shows that $(W_p, W_d) \in \omega_i \cup \omega_{iv} \setminus \omega_i \cap \omega_{iv}$ cannot be optimal either. We further rule out $\omega_v \equiv \{(W_p, W_d) : W_p \leq 2C_p/b, W_d > 0\}$ in Lemma 3. Finally, we show that $S^* = C_d$ is not optimal when ℓ is sufficiently large.

First of all, note that the $EPA's$ associated with $S^* = \bar{S}$ and C_d are both constants (i.e., independent of (W_p, W_d)), and thus the corresponding social costs are constants as well.[21] Lemma 1 shows that any decoupling rates such that $S^*(W_p, W_d) = \bar{S}$ cannot be optimal.

Lemma 1. *Any decoupled liability in the set, $\omega_i \cap \omega_{ii}$, can never attain the minimal social cost.*

Proof. From Figure 2, we see that the indicated set, $\omega_i \cap \omega_{ii}$, is the boundary at which $\{(W_p, W_d) : S^*(W_p, W_d) = \widehat{S},\ S^*(W_p, W_d) = \bar{S}\}$. Assuming $S^* = \widehat{S}$, we want to show that the partial derivative of the social cost with respect to W_p is positive. If the assertion is true, the social planner can lower W_p to reduce the social cost.

The condition $W_p = W_d + (C_p + C_d)/b$ implies that

$$\widehat{q} = b, \quad \frac{\partial EPA}{\partial W_p} = W_d \left(1 - \frac{\widehat{q}}{b}\right) \frac{\partial \widehat{q}}{\partial W_p} = 0, \quad \text{and} \quad \frac{\partial \widehat{q}}{\partial W_p} = \frac{b}{2W_d - W_p}.$$

[21] The $EPA's$ are $C_d + bW_d/2$ and C_d for $S^* = \bar{S}$ and C_d, respectively. Solving (8) implies the corresponding care levels, which then determine the social costs under these scenarios.

It follows that

$$\frac{\partial}{\partial W_p}\Big(\text{social cost}\Big) = p(c^*)\frac{(C_p + C_d)}{b}\frac{\partial \widehat{q}}{\partial W_p} > 0,$$

which completes the proof. ■

The next lemma shows that in the optimal decoupling system, the plaintiff's credibility constraint must be marginally binding so that $q' = \widehat{q}$.

Lemma 2. *For $(W_p, W_d) \in \omega_i \cup \omega_{iv}$, the necessary condition for (W_p, W_d) to solve (9) is that $(W_p, W_d) \in \omega_i \cap \omega_{iv}$.*

Proof. The lemma follows directly from Proposition 4. Essentially, one can always improve efficiency of the decoupling system by maintaining the care level while reducing the lawsuits, as long as (W_p, W_d) is not on the boundary, $\omega_i \cap \omega_{iv}$. ■

We have been focusing on the scenario of $W_p > 2C_p/b$ in which the plaintiff's case has merit. If the plaintiff's trial reward is so low that $W_p < 2C_p/b$, there will be no lawsuits at all (see Proposition 1). There is no settlement either, since the plaintiff's case has no merit. In this scenario, the injurer will set his care level at zero. The corresponding social cost is $p(0) \cdot \ell$. In contrast, when $S^* = C_d$, the case is always settled out of court. The corresponding social cost is given by $c_1 + p(c_1) \cdot \ell$ where c_1 is the solution to $1 + p'(c) \cdot C_d = 0$. We want to show that the social cost in the latter scenario is lower than that in the former.

Lemma 3. *Suppose $C_d < \ell$. Comparing the social costs in the two scenarios in which there are no lawsuits, one obtains a lower social cost when the case has merit than that when the case has no merit:*

$$c_1 + p(c_1) \cdot \ell < p(0) \cdot \ell.$$

Proof. Let $\tilde{c}$ denote the first-best level of care that minimizes the social cost without any lawsuits: $\tilde{c} \equiv \arg\min_c c + p(c) \cdot \ell$. $\tilde{c}$ satisfies the first order condition, $1 + p'(\tilde{c}) \cdot \ell = 0$. Comparing this condition with $1 + p'(c_1) \cdot C_d = 0$, one concludes that $c_1 < \tilde{c}$, provided $C_d < \ell$. Since $c + p(c) \cdot \ell$ is a convex function of c and minimized at $\tilde{c}$, the fact that $0 < c_1 < \tilde{c}$ implies $c_1 + p(c_1) \cdot \ell < p(0) \cdot \ell$. ■

It is now clear that the social planner must choose between two systems of decoupled liability. In the first option, the social planner selects $(W_p, W_d) \in \omega_{iii}$ to implement c_1 so that $S^* = C_d$, and the parties involved in an accident always settle the case out of court. The corresponding social cost is $c_1 + p(c_1) \cdot \ell$. The chance of accidents in this case is higher than desired, but there is no litigation cost. The second candidate is suggested by Lemma 2, where $(W_p, W_d) \in \omega_i \cap \omega_{iv}$ so that $S^* = \widehat{S} = S'$. The probability of litigation in this scenario is positive, but the potential injurer is more careful. The next lemma shows that the second option is more efficient when the accident loss ℓ is sufficiently large. This completes our proof of Proposition 5.

Lemma 4. *Suppose Assumptions 1 and 2 hold. There exists $\underline{\ell}$ such that for $\ell > \underline{\ell}$, the minimal social cost attained by $(W_p, W_d) \in \omega_i \cap \omega_{iv}$ is lower than $c_1 + p(c_1) \cdot \ell$.*

Proof. Consider the optimization program in (10). The first order condition can be rewritten as

$$\left(1 + p'(c^*) \cdot \left(\ell + \frac{q^*}{b}(C_p + C_d)\right)\right) \frac{dc^*}{dW_d} + p(c^*) \cdot \frac{C_p + C_d}{b} \cdot \frac{dq^*}{dW_d} = 0, \quad \text{(A3)}$$

where q^* and c^* follows from the constraints in (10). The first-order derivatives in (A3) are given by

$$\frac{dc^*}{dW_d} = \frac{p'(c^*)^2}{p''(c^*)} \cdot \frac{b}{2}\left(1 - \frac{q^*}{b} + \left(\frac{q^*}{b}\right)^2\right), \quad \frac{dq^*}{dW_d} = \frac{C_d - C_p}{2W_d^2}. \quad \text{(A4)}$$

From (A3), one observes that $\frac{dW_d}{d\ell}$ has the same sign as $\frac{dc^*}{dW_d}$, which is positive. Furthermore, the optimal W_d has no upper bound when ℓ approaches infinity, which implies $\lim_{W_d \to \infty} \frac{dq^*}{dW_d} = 0$. Assuming otherwise, then EPA and hence c^* are bounded as well, which implies that both $\frac{dc^*}{dW_d}$ and $\frac{dq^*}{dW_d}$ are finite and bounded away from zero. However, since $p'(c^*) = -\frac{1}{EPA}$ is bounded, $1 + p'(c^*) \cdot \left(\ell + \frac{q^*}{b}(C_p + C_d)\right)$ will diverge as ℓ goes to infinity, and therefore (A3) cannot hold, a contradiction. In sum, the first order condition (A3) reduces to $1+p'(c^*)\cdot\left(\ell + \frac{1}{2}(C_p + C_d)\right) = 0$ in the limit, assuming that $\frac{p'(c)^2}{p(c)p''(c)}$ is bounded away from zero, which is implied by Assumption 2.

Let c_2 be the care level that corresponds with the approximate solution: c_2 solves the first order condition $1 + p'(c_2) \cdot (\ell + \frac{1}{2}(C_p + C_d)) = 0$. The imputed social cost is given by $c_2 + p(c_2) \cdot (\ell + \frac{1}{2}(C_p + C_d))$. We want to show that this cost is lower than $c_1 + p(c_1) \cdot \ell$. The comparison reduces to

$$\frac{1}{2}(C_p + C_d) \cdot p(c_2) < -\frac{p(c_2) - p(c_1)}{c_2 - c_1} \cdot \ell.$$

The left hand side converges to zero, provided Assumption 1. The right hand side is greater than $-p'(c_2) \cdot \ell$ from the Mean-Value Theorem and the fact that $p''(c) > 0$. By the definition of c_2, $-p'(c_2) \cdot \ell$ is equal to $\frac{\ell}{\ell + \frac{1}{2}(C_p + C_d)}$, which converges to 1. Thus, the inequality holds when ℓ is large enough. ■

Proof of Corollary 7

From (A4), one observes that $\frac{dc^*}{dW_d}$ is positive, while the sign of $\frac{dq^*}{dW_d}$ depends on that of $C_d - C_p$. Consider first the case of $C_d < C_p$. It follows that $\frac{dq^*}{dW_d} < 0$, and hence $1 + p'(c^*) \cdot \left(\ell + \frac{q^*}{b}(C_p + C_d)\right) > 0$ from (A3). Comparing this inequality with the injurer's first order condition, one obtains

$$\ell + \frac{q^*}{b}(C_p + C_d) < q^* \left(1 - \frac{q^*}{2b}\right) W_d + C_d.$$

$\frac{q^*}{b}(C_p + C_d) - C_d$ is positive since $q^* > \frac{b}{2}$ and $C_d < C_p$. Moreover, $q^* \left(1 - \frac{q^*}{2b}\right)$ is bounded above by $\frac{b}{2}$ as $q^* \leq b$. Thus, $W_d > \frac{2\ell}{b}$ if $C_d < C_p$.

Next, suppose $C_d > C_p$. Recall that $\tilde{c}$ is the first-best level of care: $1 + p'(\tilde{c}) \cdot \ell = 0$ (see the proof of Lemma 3). Define $\tilde{W}_d$ and $\tilde{q}$ such that they implement $\tilde{c}$ according to (8): $\tilde{q}(1 - \frac{\tilde{q}}{2b})\tilde{W}_d + C_d = \ell$ and $\tilde{q} = \frac{b}{2} - \frac{C_d - C_p}{2\tilde{W}_d}$. The first equation implies that $\tilde{W}_d$ diverges as ℓ goes to infinity, and therefore $\lim_{\ell \to \infty} \tilde{q} = \frac{b}{2}$ from the second equation. We want to show that the derivative of the social cost evaluated at $\tilde{W}_d$ is negative, which will imply that the optimal W_d is greater than $\tilde{W}_d$. Substituting the equations that define $\tilde{W}_d$ and $\tilde{q}$ into the left hand side of (A3), the desired inequality reduces to

$$\frac{(1 - \frac{2\tilde{q}}{b})(1 - \frac{\tilde{q}}{2b})}{1 - \frac{\tilde{q}}{b} + (\frac{\tilde{q}}{b})^2} < \frac{p'(\tilde{c})^2}{p(\tilde{c})p''(\tilde{c})} \cdot \frac{\ell - C_d}{\ell}.$$

The left hand side converges to zero when $\tilde{q}$ converges to $\frac{b}{2}$, or, equivalently, when ℓ approaches infinity. Provided Assumption 2, the limit of the right hand side is greater than one, and hence the inequality holds. It follows that the optimal W_d is greater than $\tilde{W}_d$, which is equal to $\frac{\ell - C_d}{\tilde{q}(1 - \tilde{q}/2b)}$. The last term is bounded below by $\frac{8(\ell - C_d)}{3b}$ since $\tilde{q}$ is less than $\frac{b}{2}$ in the current scenario. ■

Proof of Corollary 8

From the first constraint in (10) ($q^* = \frac{b}{2} - \frac{C_d - C_p}{2W_d}$), one observes that the settlement rate in equilibrium, $1 - \frac{q^*}{b}$, is greater than $\frac{1}{2}$ if and only if $C_p < C_d$. Furthermore, $C_p < C_d$ implies that $\frac{dq^*}{dW_d} > 0$ in view of (A4), and therefore $1 + p'(c^*) \cdot \left(\ell + \frac{q^*}{b}(C_p + C_d)\right)$ in (A3) is negative since $\frac{dc^*}{dW_d}$ is always positive. Comparing this inequality with the injurer's first order condition, $1 + p'(c^*)EPA = 0$, one obtains

$$-p'(c^*) \cdot \left(\ell + \frac{q^*}{b}(C_p + C_d)\right) > -p'(c^*) \cdot \left(q^* \left(1 - \frac{q^*}{2b}\right) W_d + C_d\right).$$

The inequality states that the injurer's marginal benefit from exerting care is lower than the marginal benefit for the society. It means that the injurer should have raised his care level from the society's point of view. ■

Proof of Proposition 9

In the proof of Lemma 4, we have shown that $\frac{dW_d}{d\ell}$ and $\frac{dc^*}{d\ell}$ are both positive. The settlement demand S^* in equilibrium is equal to $\frac{b}{2}W_d + \frac{C_p+C_d}{2}$, which is proportional to W_d. Hence, $\frac{dS^*}{d\ell}$ is positive as well. Meanwhile, the changes in W_p and q^* induced by an increase of ℓ depend on the slope of the boundary $\omega_i \cap \omega_{iv}$, which is determined by the sign of $C_d - C_p$. ■

Proof of Example 1

With $C_p = C_d$, W_p is a constant on the path of $\omega_i \cap \omega_{iv}$. The constraints in (10) reduces to $q^* = \frac{b}{2}$ and $1+p'(c^*)\cdot\left(\frac{3b}{8}W_d + C_d\right) = 0$. The social problem is to minimize $c^* + p(c^*)\cdot\left(\ell + \frac{1}{2}(C_p + C_d)\right)$, which implies $(1 + p'(c^*)\cdot(\ell + C_d))\frac{dc^*}{dW_d} = 0$ as the first order condition. Comparing this condition with the previous one, one obtains $\frac{3b}{8}W_d + C_d = \ell + C_d$, and hence $W_d = \frac{8\ell}{3b}$.

Assuming $p(c) = \exp(-c)$, the minimal social cost attained in (10) is $\ln(\ell + C_d) + 1$. Meanwhile, the social cost associated with $S^* = C_d$ is given by $\ln C_d + \ell/C_d$. The latter cost is greater than the former if $\ell/C_d > 2.146$. ■

Proof of Example 2

First of all, the social cost associated with $S^* = C_d$ is not affected by endogeneity of $F(q)$, and thus is given by $\ln C_d + \ell/C_d$.

Assuming $(W_p, W_d) \in \omega_{iv}$, with $p(c) = b(c) = e^{-c}$, the first order condition to the injurer's problem reduces to $1 + p'(c^*)\cdot(q^*W_d + C_d) = 0$, which implies $c^* = \ln(q^*W_d + C_d)$. The care level here is higher than the level when $F(q)$ is exogenous. In other words, given the same decoupling rates, the injurer will take extra care if being more careful helps to reduce his share of responsibility. Provided $C_p = C_d$, one obtains $q^* = \frac{b}{2}$ from (10), and thus the social planner's problem is to minimize $c^* + p(c^*)\cdot\left(\ell + \frac{1}{2}(C_p + C_d)\right)$. The first order condition is $(1 + p'(c^*)\cdot(\ell + C_d))\frac{dc^*}{dW_d} = 0$. Comparing this condition to the previous one, one concludes that $W_d = \frac{\ell}{q^*} = \frac{2\ell}{b}$, where

$b = \exp(-c^*) = \frac{1}{\ell + C_d}$. ■

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