A Strategic Theory of Antitrust Enforcement

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Abstract. We develop a strategic model of private and public enforcement of the antitrust laws. The model highlights the tradeoff that private firms are more likely than the government to be informed about actual antitrust violations, but are also more likely to use the antitrust laws strategically, to the disadvantage of consumers. With coupled damages (plaintiff receives what defendant pays), if the court is sufficiently accurate, then adding private enforcement to public enforcement always increases social welfare, while if the court is less accurate, then it increases welfare only if the government is sufficiently inefficient in litigation. Moreover, pure private enforcement always yields weakly lower welfare than private enforcement combined with public enforcement. However, in general, achieving the welfare-maximizing outcome requires private enforcement with damages that are both multiplied and decoupled.

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I. Introduction

VeriSign is the official registrar of both .com and .net internet addresses. In September 2003, Verisign began redirecting mistyped internet addresses to its own sitefinder.com advertising site. Many internet service providers objected to this practice and asked ICANN, the Internet Corporation for Assigned Names and Numbers, to prevent Verisign’s redirection. ICANN studied the matter and concluded that Verisign’s redirections had undesirable side effects and banned the redirection. Verisign sued that ICANN’s determination was an illegal conspiracy under the antitrust laws. The 16 page decision of Judge A. Howard Matz concluded that the case was so deficient as to not even merit a trial. Meanwhile, Popular Enterprises, which buys expired domains and redirects them to its netster.com advertising site, sued Verisign, again on antitrust grounds, for offering sitefinder.com in the first place. The two suits concern the same action (sitefinder.com) and one says banning it is an antitrust violation, while the other says offering it is an antitrust violation. At least one and perhaps both of these suits are in fact meritless. Is it in the public interest to permit private antitrust litigation?

In the United States, Section 7 of the Sherman Act of 18901 and Section 4 of the Clayton Act of 19142 entitle any firm to bring a lawsuit against a competitor for three times the damages suffered from any violation of the antitrust laws. Private enforcement of the antitrust laws is therefore explicitly permitted and encouraged, and supplements public enforcement by the Antitrust Division of the U.S. Department of Justice and the Federal Trade Commission.

From 1970 to 1995, private lawsuits have outnumbered public lawsuits by a 9 to 1 ratio.3 The disproportionate number of private actions may be cause for concern. In many private cases, firms bring suit against their competitors. Firms might attempt to use the antitrust laws to win in the courts what they were unable to win in honest competition with their rivals. In other words, they might strategically abuse of the antitrust laws.

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1 An Act to Protect trade and commerce against unlawful restraints and monopolies, ch. 647, 26 Stat. 209 (1890) [current version at 15 U.S.C. § 1 et seq. (2003)].
example, firms might sue under the antitrust laws to prevent their rivals from competing vigorously, or to prevent large potential competitors from entering their market.

However, while private firms tend to have the greatest incentives to take enforcement actions, they also tend to have the best information about the case. Private firms are generally much better informed than public regulators about their own activities and those of their close competitors. “For a regulator to obtain comparable information would often require virtually continuous observation of parties’ behavior, and thus would be a practical impossibility” (Shavell, 1984, p. 360).

In this paper, we develop a strategic model of antitrust enforcement that highlights the tradeoff that firms are more likely than the government to be informed about antitrust violations, but are also more likely to use the antitrust laws strategically, to the detriment of consumers. Firms choose whether or not to take an action that either violates the antitrust laws or improves their own efficiency, rival firms choose whether or not to sue them privately, and the government chooses whether or not to sue them publicly. The rival firms are perfectly informed about whether an illegal or efficient action is taken, but the government is not. The government only wants to sue if an illegal action is taken, but rival firms may want to sue even if an efficient action is taken. The model is solved for its equilibrium outcomes under different enforcement mechanisms (including pure private, pure public, and public and private enforcement), which are then compared in terms of social welfare.

Assuming that damages are coupled, in the sense that the plaintiff receives what the defendant pays, we find that adding private enforcement to public enforcement is always socially beneficial if the court is sufficiently accurate. In this case, firms never strategically abuse of the laws, only suing when their competitors have committed an antitrust infraction, so that private enforcement only serves to counter antitrust harm, and hence improves welfare. But if the court is less accurate, adding private enforcement to public enforcement is beneficial only if the government is sufficiently inefficient in litigation. In this case, firms always sue when their rivals take efficient actions, preferring to take a chance with the courts than suffer a certain loss in market share. Society benefits from private suits only if the public enforcement is sufficiently poor and the legitimate
private suits outweigh the strategic suits. This requires poor public enforcement since otherwise most of the legitimate suits are brought by the government.

We also find that the combination of private and public enforcement tends to lead to a greater probability of private than public lawsuit. In most cases, firms have sufficient incentive to sue if they learn that their rivals have actually violated the antitrust laws. Knowing this, the government has no reason to sue, since it can expect that most of the rightful suits are already being initiated privately. Thus, public enforcement usually gives way to private enforcement when the two are in play. This is consistent with the observation that private suits have largely outnumbered public suits in the United States.

In the model, the government only has reason to sue in the case where the litigation costs of private firms are very high. But in this case, firms never sue, even if they know that their rivals would otherwise get away with an antitrust violation, so that pure private enforcement yields lower welfare than public enforcement, whether or not it is combined with private enforcement, as long as society prefers some public enforcement to no enforcement at all. Moreover, in the majority of cases, where the government has no reason to sue, pure private enforcement is equivalent to private and public enforcement, which yields lower welfare than pure public enforcement if the government is sufficiently efficient in litigation. Hence, pure private enforcement is a weakly dominated mechanism, and in particular, it yields weakly lower social welfare than the mechanism of private and public enforcement.

In general, however, the social welfare-maximizing outcome is achieved only through a mechanism with private enforcement and damages that are multiplied and decoupled. If the plaintiff wins at trial, the defendant can be required to pay the plaintiff a multiple of the amount of damages that it inflicted on the plaintiff by violating the antitrust laws. Moreover, the plaintiff can be made to receive only a fraction of what the defendant pays for an antitrust violation, with the rest going to society. Multiplying damages prevents firms from taking illegal actions, while decoupling them reduces the incentives of firms to strategically abuse of the antitrust laws. Therefore, under private enforcement with damages both judiciously multiplied and decoupled, firms always take legal actions and never take illegal ones—the overall social welfare-maximizing outcome.
The remainder of the paper is organized as follows. Section II situates the contribution within existing literature on private enforcement of public law. Section III develops the strategic model of antitrust enforcement. Section IV solves the model under pure public enforcement. Section V solves it under private and public enforcement, and under pure private enforcement. Section VI compares mechanisms in terms of social welfare. Section VII derives the socially optimal mechanism. Section VII summarizes the results, draws policy implications, and suggests avenues for further research.

II. Related Literature

The merits of private versus government enforcement of agreements and laws is a central issue in the theory of government. Before the development of government, people had to rely exclusively on private enforcement of law, which according to Thomas Hobbes (1968, p. 186) was one of the main reasons that “the life of man [was] solitary, poor, nasty, brutish, and short.” Hobbes explained that rational self-interest would lead individuals to willingly forego their ability to enforce law for a public monopoly over the use of physical force, because this would allow them to spend a smaller fraction of their resources on self-protection and a larger fraction on other more productive activities.

Max Weber (1968), who formally defined the state as the entity that possesses a monopoly on the legitimate use of physical force, further developed the Hobbesian argument in favor of pure public enforcement. He argued that public enforcers would enforce the law most effectively because they are subject to hierarchical control and cannot be motivated by vengeance, unlike private enforcers. Moreover, public enforcers would operate on a professional, full-time basis, which would allow them to exploit scale economies and develop specialized expertise, whereas private enforcers would only operate on a personal, part-time basis.

Our finding that pure private enforcement of the antitrust laws is generally not socially optimal is broadly consistent with the Hobbesian argument. Moreover, broadly consistent with Weber’s argument is our finding that adding private enforcement to public enforcement is socially harmful if the government is sufficiently efficient in litigation. However, the arguments by Hobbes and Weber are of much broader scope, and
not limited to antitrust. Our analysis can be viewed as an application of these weighty ideas to antitrust analysis.

More recently, economists and legal scholars have offered another perspective on private law enforcement. Olsen (1965) argues that private enforcers are less susceptible to capture by special interests than public enforcers, if only because the former tend to largely outnumber the latter. Cohen and Rubin (1985) argue that law enforcement may not be the only objective of public enforcers; they may also be interested in maximizing their political support. Becker and Stigler (1974) argue that free competition among private law enforcers for the damages that are levied against convicted violators could achieve deterrence as efficiently as optimal public enforcement.

Landes and Posner (1975), Schwartz (1981), and Posner (1992) challenge the Becker and Stigler argument. Under public enforcement, if the probability of enforcement is equal to one, the penalties should be set equal to the social costs of the illegal activity. By increasing the penalties and reducing the probability of enforcement, society can achieve the same level of deterrence at less cost. However, under private enforcement, increasing the penalties increases the probability of enforcement, as it increases the incentives of private enforcers. Thus, private enforcement can lead to over-deterrence from the social standpoint.

Polinsky (1980) challenges the argument that private enforcement always leads to over-deterrence. He argues that private parties would only be willing to invest in deterrence if their enforcement costs are less than their enforcement revenues. But their revenues are limited by the net wealth of potential violators. If potential violators are poor, private parties could only achieve large revenues by engaging in a lot of deterrence. Therefore, if enforcement costs are also large, private parties may not find it worthwhile to invest in deterrence. However, potential criminals know whether they have a lot of assets on the line, and potential plaintiffs can usually readily observe the assets. Consequently, rich criminals are deterred, poor ones are not; there is still overdeterrence of rich criminals even if limited liability produces underdeterrence of poor criminals.

These arguments about over-deterrence generally assume that private enforcers act legitimately, that is, that they never seek to enforce the law against individuals who have not engaged in the illegal activity. In reality, potential private enforcers may have
incentive to behave strategically. This danger is particularly high in the antitrust field because the plaintiffs are often competitors or takeover targets of defendants. They are likely to employ private enforcement strategically, that is, they are likely to sue even if they know that their competitors did not violate the antitrust laws.

The prevalence of strategic abuse of the antitrust laws by private firms is documented by, among others, Baumol and Ordover (1985), Breit and Elzinga (1985), Shughart II (1990), Brodley (1995), McAfee and Vakkur (2004), and McAfee et al. (2005). Not only are the antitrust laws used by firms to prevent large competitors from entering their market or existing rivals from competing vigorously, but they are also used to extort funds from successful rivals, improve contractual conditions, enforce tacit collusive agreements, respond to existing suits, and prevent hostile takeovers. For instance, hostile takeover targets often initiate antitrust lawsuits against their predators, because these lawsuits create substantial delays that allow the target firms to implement various anti-takeover strategies, like poison pills. If the intended takeover is good for the market, these antitrust actions have a negative effect. The different strategic uses of the antitrust laws often have little to do with promoting social efficiency.

The extent to which firms strategically abuse of the antitrust laws under private enforcement also depends crucially on the structure of damage awards in private antitrust cases. The welfare effects of multiplying, and in particular trebling, damages are discussed by Breit and Elzinga (1974, 1985), Block et al. (1981), Easterbrook (1985), Salop and White (1986), Newmark (1988), and Briggs et al. (1996). Treble damages reduce the incentives of firms to violate the antitrust laws, but also increase their incentives to use the antitrust laws strategically against their rivals.

For example, firms can use the powerful threat of treble damages to extort funds from successful rivals. The actions that are taken to extort money are often resolved through the payment of a “tax on success” for the firms whose positions are sought after by competitors. But taxes on success discourage investment and innovation, which harms consumers. In our model, the welfare-maximizing outcome cannot in general be achieved under private enforcement only by multiplying damages, precisely because multiplying

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4 These ways to use the antitrust laws strategically are explained in detail, and their use is documented with numerous U.S. court cases, in McAfee and Vakkur (2004) and McAfee et al. (2005).
damages encourages firms to sue their successful rivals, which in turn discourages their rivals from taking efficiency-improving actions.

Treble damages can also have perverse effects on antitrust enforcement by consumers. Block et. al. (1981) develop a model of the interaction between collusive sellers and competitive buyers under treble damages for price-fixing, which Salant (1987) modifies to take into account that treble damages may have the perverse effect of stimulating demand at any given price set by the cartel. The idea is that buyers have incentive to “get damaged” if they expect to collect treble damages later. However, this perverse effect is weaker if buyers are imperfectly informed about the cartel’s costs (Besanko and Spulber, 1990). While these models concern enforcement by consumers, our model concerns enforcement by competing firms.

In theory, private damage awards can also be decoupled, in the sense that the amount awarded to the plaintiff differs from the amount paid by the defendant. This idea was introduced by Schwartz (1980). The welfare effects of decoupling are analyzed informally by Polinsky (1986), and formally by Polinsky and Che (1991). Polinsky argues that for any damage multiplier, there is a damage decoupler that achieves the same deterrence level at lower cost. Consider a damage multiplier that achieves a given deterrence level. Increase what the defendant pays, which increases deterrence by increasing the penalty in case of detection. Then reduce what the plaintiff receives, which reduces deterrence by reducing enforcement and thus the detection probability, until the same level of deterrence is achieved as under the original damage multiplier. This deterrence level is achieved with less enforcement, and thus at lower cost.

Polinsky and Che formalize this argument in a standard tort model. In their model, decoupling reduces the plaintiff’s incentive to sue, thereby reducing litigation costs, without sacrificing the potential defendant’s incentive to exercise care. Decoupling has a similar beneficial effect in our model of antitrust enforcement, although our model is conceptually different from the one analyzed by Polinsky and Che, as they do not consider strategic interaction or the possibility of strategic abuse of the law. In our model, sufficiently multiplying damages ensures that illegal actions are always deterred, while sufficiently decoupling them eliminates strategic abuse of the law and thereby ensures
that efficient actions are never deterred, so that private enforcement may achieve the first-best outcome.

III. Strategic Model of Antitrust Enforcement

Consider an industry comprised of two competing firms, which we call firm 1 and firm 2, and a government agency, which we call GOV, in charge of enforcing the antitrust laws on behalf of the public. Initially, Nature presents firm 1 with a potential action, $A$. With probability $p$, the action reduces firm 1’s own cost, and with probability $1 - p$, the action increases firm 2’s cost. Whether the action is own cost reducing ($RC_1$) or rival cost increasing ($IC_2$) is the type of the change. An action of type $IC_2$ violates the antitrust laws, harming both competition and consumers, while an action of type $RC_1$ does not, because it is beneficial to consumers. Knowing the true type, firm 1 decides whether or not to act, denoted by $A$ or $\neg A$, respectively. If firm 1 does not take the action, the game ends, and firm 1, firm 2, and GOV receive payoffs normalized to zero.

If firm 1 takes an action that reduces its own cost or increases firm 2’s cost, then an amount $T$ of profits is transferred from firm 2 to firm 1, and the new market shares are realized. GOV then gets a noisy signal of whether firm 1’s cost fell or firm 2’s cost rose. If firm 1’s cost fell, then with probability $q$, GOV gets a signal that firm 1’s cost fell ($SRC_1$), and with probability $1 - q$, GOV receives a signal that firm 2’s cost rose ($SIC_2$). If firm 2’s cost rose, then with probability $q$, GOV receives the signal $SIC_2$, and with probability $1 - q$, it receives the signal $SRC_1$. We assume that $q > 1/2$, that is, GOV’s signal is more often right than wrong, but not always right.

After receiving the signal, GOV decides whether or not to sue firm 1 for violating the antitrust laws, $S$ or $\neg S$. Firm 1’s litigation cost is $L_1$, and GOV’s litigation cost is $L_{GOV}$. We assume that GOV never sues when the signal indicates that firm 1’s cost fell. If GOV sues, then the case proceeds to trial. We assume that the court outcome is exogenous. If firm 1’s cost fell, then with probability $r$, the court finds firm 1 innocent ($I$), and with probability $1 - r$, it finds firm 1 guilty ($G$). If firm 2’s cost rose, then with probability $r$, the court renders the verdict $I$, and with probability $1 - r$, it renders verdict
The court outcome is correlated with the truth, but imperfect. More precisely, \( r > 1/2 \).

If firm 1 is found guilty, then the court orders firm 1 to undo its action and pay compensatory damages to firm 2 in the exact amount \( T \).

If firm 1 is correctly found guilty of violating the antitrust laws, social welfare increases by an amount \( x \) due to the court’s intervention. If firm 1 is incorrectly found guilty, social welfare decreases by the amount \( x \). If firm 1 takes an action that reduces its own cost or increases firm 1’s cost, GOV sues firm 1, and the court finds firm 1 innocent, then the payoffs of firm 1, firm 2, and GOV are \( T - L_1, -T, \) and \( x - L_{GOV} \), respectively. But if the court finds firm 1 guilty, then their respective payoffs are \( -L_1, 0, \) and \( -L_{GOV} \).

The value of \( x \) is determined by the harm to competition created by an anticompetitive action, or the loss from preventing a cost decrease. The actual value should be determined by the competitive model that is relevant to the industry. If the industry competition were Cournot, with linear demand \( p = 1 - Q \), and constant marginal costs, the market price would be \( p = (c_1 + c_2 + 1)/3 \) and the value of consumer surplus is \( (1 - p)^2 / 2 \). This makes calculating the efficiency change of an unnecessary cost increase or lost cost decrease straightforward. The value of \( T \), which is lost earnings by firm 2 given the action of firm 1, is also readily computed in this model. Similarly, if competition is with differentiated products, the value of \( x \) is interpreted to be the welfare change associated with an unnecessary cost increase to one of the firms, or a foregone cost decrease. While in principle the effect of a cost increase for firm 2 or a decrease for firm 1 could be different, we take them to be the same for simplicity.

If GOV does not sue (either because it received the signal that firm 1 reduced its own cost, or chose not to sue even though it received the signal that firm 1 increased its rival’s cost), then firm 2 chooses whether or not to sue firm 1 privately, \( S \) or \( \neg S \). Unlike GOV, firm 2 is fully informed about the type of firm 1’s action. Firm 2’s litigation cost is \( L_2 \). If firm 2 does not sue, then the game ends, and the payoffs of firm 1, firm 2, and GOV are \( T, -T, \) and \( x \), respectively. If firm 1 sues, then the case proceeds to trial. For simplicity, we assume that the court outcome is the same as when GOV sues. That is, the court’s decision is not conditioned on the identity of the plaintiff. With probability \( r \), the
court renders the correct verdict. Verdict $G$ results in a transfer $T$ from firm 1 to firm 2, which increases welfare by $x$ if firm 1 is guilty, and reduces it by $x$ if firm 1 is not guilty.

**INSERT GAME TREE.**

If firm 1 takes an action that reduces its own cost or increases firm 1’s cost, GOV does not sue firm 1, but firm 2 sues firm 1 privately, and the court finds firm 1 innocent, then the payoffs of firm 1, firm 2, and GOV are $T - L_1$, $-T$, and $x - L_{GOV}$, respectively. If the court finds firm 1 guilty in this case, then their respective payoffs are $-L_1$, 0, and $-L_{GOV}$. We assume firm 1’s litigation costs are greater than firm 2’s litigation costs (because it costs more to defend against a lawsuit than it does to carry one out against a rival), that is, $L_1 > L_2$.

This completes the description of the benchmark game between GOV, firm 1, and firm 2. We are interested in determining the welfare effects of allowing private antitrust action. To do so, we compare the outcomes of the benchmark game, in which both firm 2 and GOV can sue firm 1 (private and public enforcement), to those of the reduced games, in which only GOV can sue firm 1 (pure public enforcement) and in which only firm 2 can sue firm 1 (pure private enforcement). In the pure public enforcement game, if GOV does not sue, the game ends and payoffs are realized. The pure public enforcement and benchmark games are otherwise identical. Similarly, the benchmark and pure private enforcement games are identical except that GOV cannot sue in the latter.

**IV. Public Enforcement**

Consider the reduced game where only GOV can sue firm 1. Consider first GOV’s decision. Let $\gamma_1$ be the probability that firm 1 takes the action to lower its own cost, and $\gamma_2$ be the probability that firm 1 takes the action to increase firm 2’s cost. Suppose GOV receives the signal $SIC_2$. Then GOV’s expected utility from $S$ is:

\[
B(-L_{GOV} + rx) + (1 - B)(-L_{GOV} - (1 - r)x).
\]
where \( B = \frac{\frac{\gamma_1(p)(1-q)}{p(1-q) + \gamma_2(1-p)q}}{\gamma_1(p)(1-q) + \gamma_2(1-p)q} \). GOV’s expected utility from \( \neg S \) is:

\[
(2) \quad B(x) + (1 - B)(-x).
\]

GOV is indifferent between \( S \) and \( \neg S \) if

\[
(3) \quad rx - Bx - L_{GOV} = 0.
\]

In this case, GOV randomizes. If (3) is positive, GOV chooses \( S \), and if it is negative, GOV chooses \( \neg S \).

Let us now consider firm 1’s decision. Let \( \sigma_G \) be GOV’s lawsuit probability given that it receives a signal that firm 1 took an action to increase its rival’s cost. Suppose firm 1 learns that the type is \( RC_1 \). Then firm 1 randomizes between \( A \) and \( \neg A \) when:

\[
(4) \quad T - (1 - q)\sigma_G((1 - r)T + L_A) = 0 \iff \sigma_{Ga} = \frac{T}{(1 - q)((1 - r)T + L_A)}.
\]

If (4) is positive, firm 1 chooses \( A \) given \( RC_1 \), and if it is negative, firm 1 chooses \( \neg A \). When firm 1 learns that the type is \( IC_2 \), firm 1 randomizes between \( A \) and \( \neg A \) when:

\[
(5) \quad T - q\sigma_G(rT + L_A) = 0 \iff \sigma_{Gb} = \frac{T}{q(rT + L_A)}.
\]

If (5) is positive, firm 1 chooses \( A \) given \( IC_2 \), and if it is negative, firm 1 chooses \( \neg A \).

Using the best-response conditions in (3), (4), and (5), we can derive the equilibria and corresponding welfare values of the game with pure public enforcement. We measure welfare as the sum of the two players’ equilibrium payoffs. The results are presented in the following proposition.
Proposition 1. The stable Nash equilibria of the game with pure public enforcement.

(i) If \( T < q(rT + L_1) \) and \( \left( r - \frac{p(1-q)}{p(1-q) + (1-pq)} \right) x > L_{GOV} \), then the unique stable equilibrium is \( \gamma_1^* = 1 \), \( \gamma_2^* = \frac{p(1-q)}{(1-p)q + (1-pq)} \in (0,1) \), \( \sigma_G^* = \frac{T}{q(rT + L_1)} \in (0,1) \), and equilibrium welfare is

\[
W^* = px - px\left( \frac{p(1-q)}{p(1-q) + (1-pq)} \right) \left( \frac{L_{GOV} + L}{rx - L_{GOV}} \right).
\]

(ii) If \( T > q(rT + L_1) \) and \( \left( r - \frac{p(1-q)}{p(1-q) + (1-pq)} \right) x > L_{GOV} \), then \( \gamma_1^* = \gamma_2^* = 1 \), \( \sigma_G^* = 1 \), and

\[
W^* = px - p(1-q)(1-r)x - (1-p)(1-q)r x - (p(1-q) + (1-p)q)(L_1 + L_{GOV}).
\]

(iii) If \( \left( r - \frac{p(1-q)}{p(1-q) + (1-pq)} \right) x < L_{GOV} \), then \( \gamma_1^* = \gamma_2^* = 1 \), \( \sigma_G^* = 0 \), and

\[
W^* = px - (1-p)x.
\]

Proof. Propositions are proved in the Mathematical Appendix.

If GOV’s litigation costs are so high that it would not want to sue even if it knew that firm 1 always takes the illegal action (region iii), then GOV never sues, and firm 1 always takes the action, whether or not it is legal. If GOV’s litigation costs are lower, but firm 1’s litigation costs are so low that it would want to take the illegal action even if it knew that GOV always sues (region ii), then firm 1 always takes the action, whether or not it is legal, and GOV always sues.

If GOV’s litigation costs are not too high and firm 1’s litigation costs are not too low (region i), there is a Nash equilibrium in which GOV sues with positive probability, and firm 1 takes an illegal action with positive probability. The government only wants to sue if firm 1 takes an illegal action, but is not perfectly informed. Thus, it must consider the possibility that it might sue firm 1 for taking an efficient action. On the other hand, firm 1 does not want to take an illegal action if the government sues with high probability. But if firm 1 does not take an illegal action, the government does not want to sue with high probability. Therefore, the government randomizes between suing and not suing, and firm 1 randomizes between taking and not taking an illegal action.

The lower are firm 1’s litigation costs, the higher is GOV’s expectation that firm 1 has taken the illegal action if it arose, and hence the higher is GOV’s probability of suing. On the other hand, the higher are GOV’s litigation costs, the higher is firm 1’s expectation that GOV will not sue, and hence the higher is the probability that firm 1 takes an illegal action when it arises. Since GOV’s signal is noisy, it might receive the
signal that firm 1 took a legal action even if firm 1 actually took an illegal action, in which case GOV would not sue firm 1, by assumption. Thus firm 1 is less likely to take the illegal action the less noisy is GOV’s signal ($\gamma_2^*$ decreases as $q$ gets closer to 1). Similarly, since the court is not perfectly accurate, firm 1 might be acquitted even if it took the illegal action and was sued by GOV. Thus firm 1 is less likely to take the illegal action the more accurate is the court ($\gamma_2^*$ decreases as $r$ gets closer to 1). However, firm 1 still takes the illegal action with positive probability even if the court is perfectly accurate ($r = 1$), as long as GOV is not perfectly informed ($q < 1$).

There is another Nash equilibrium in region (i), in which firm 1 does not take an action, whether or not it is legal, and GOV would sue with high probability if firm 1 took an action. However, this equilibrium is not stable, in the sense that it does not survive reasonable trembles onto out-of-equilibrium strategies. If the government sues with high probability, firm 1 may be deterred from taking any kind of action. But suppose firm 1, on rare occasion, mistakenly takes an efficient action when the opportunity arises; and also on rare occasion, mistakenly takes an illegal action when it presents itself. The first type of mistake is less costly than the second, since the government sues with high probability and the court is more often right than wrong. Thus, the first type of mistake should be more common, even though both types of mistakes should be rare. If the first type of mistake is sufficiently more common than the second, then if the government ever observes an action, it would be sufficiently likely to believe that the action was efficient that it would not sue with high probability, and firm 1 would not be deterred from taking an efficient action. If attention is restricted to stable equilibria, firm 1 is never deterred from taking an efficient action under pure public enforcement.

V. Private Enforcement

We now turn to the full model where both GOV and firm 2 can sue. Consider firm 2’s decision. Let $\alpha_1$ be the probability that firm 1 takes the action when the action reduces its own cost, $\alpha_2$ be the probability that firm 1 takes the action when the action increases firm 2’s cost, and $\sigma_{G2}$ be the probability that GOV sues when it receives the signal that firm 1
took an action that increased its rival’s cost. Consider the two subgames for firm 2. Suppose the type is $RC_1$. Given $\alpha_i$ and $\sigma_{g2}$, if firm 2 chooses $S$, then its expected utility is:

$$
(6) \quad \alpha_i q(\pi_2 - L_2 - rT) + \alpha_i (1-q)(1-\sigma_{g2})(\pi_2 - L_2 - rT) + \alpha_i (1-q)\sigma_{g2}(\pi_2 - rT) + (1-\alpha_i)\pi_2.
$$

If firm 2 chooses $\neg S$, then its expected utility is:

$$
(7) \quad \alpha_i q(\pi_2 - T) + \alpha_i (1-q)(1-\sigma_{g2})(\pi_2 - T) + \alpha_i (1-q)\sigma_{g2}(\pi_2 - rT) + (1-\alpha_i)\pi_2.
$$

Therefore, firm 2 randomizes between $S$ and $\neg S$ when

$$
(8) \quad ((1-r)T - L_2)\left[\alpha_i q + \alpha_i (1-q)(1-\sigma_{g2})\right] = 0.
$$

Firm 2 chooses $S$ when (8) is positive, and $\neg S$ when it is negative. If $\alpha_i \neq 0$, firm 2 sues if and only if $T(1-r) < L_2$.

Now suppose the type is $IC_2$. Then firm 2 randomizes between $S$ and $\neg S$ when

$$
(9) \quad (rT - L_2)\left[\alpha_i (1-q) + \alpha_i q(1-\sigma_{g2})\right] = 0,
$$

chooses $S$ when (9) is positive, and chooses $\neg S$ when it is negative. When $\alpha_i \neq 0$, firm 2 sues if and only if $rT < L_2$.

Let us now consider GOV’s decision. Let $\beta_T$ be the probability that firm 2 chooses to sue given that firm 1 reduced its own cost, and $\beta_T$ the probability that firm 2 chooses to sue given that firm 1 increased firm 2’s cost, given that GOV chose not to sue firm 1. Upon receiving signal $SIC_2$, GOV’s expected utility from $S$ is:

$$
(10) \quad D(-L_{GOV} + rx) + (1-D)(-L_{GOV} - (1-r)x),
$$

where $D = \frac{\alpha_2 p(1-q)}{\alpha_2 p(1-q) + \alpha_1 (1-p)q}$. GOV’s expected utility from $\neg S$ is:
GOV randomizes between $S$ and $\neg S$ when

\[ D(\beta rx + (1 - \beta_1)x) - (1 - D)(\beta_2(1 - r)x) + (1 - \beta_2)x) . \]

GOV randomizes between $S$ and $\neg S$ when

\[ -D(1 - r)x(1 - \beta_1) + (1 - D)rx(1 - \beta_2) - L_{GOV} = 0, \]

chooses $S$ when (12) is positive, and $\neg S$ when it is negative.

If $(1 - r) < L_2 < Tr$ or $L_2 < (1 - r) < Tr$, firm 2 always sues when firm 1 takes the illegal action, that is, $\beta_1 = 1$. Knowing this, GOV’s best response is to never sue, that is, $\sigma^*_{G2} = 0$, because it can count on firm 2 to always sue when firm 1 takes the illegal action. But if $(1 - r) < Tr < L_2$, firm 2 never sues in either case, that is, $\beta_1 = \beta_2 = 0$. Knowing this, GOV’s best response is to sue with a probability $\sigma^*_{G2} \in [0, 1]$. Given firm 1’s choices, $\alpha_1$ and $\alpha_2$, GOV then randomizes if and only if

\[ D = \frac{rx - L_{GOV}}{x} \Leftrightarrow \alpha_1 = \left( \frac{rx - L_{GOV}}{(1 - r)x + L_{GOV}} \right) \frac{(1 - p)q}{p(1 - q)} \alpha_2 . \]

If firm 2 does not sue even if it knows that firm 1 violated the antitrust laws, GOV cannot count on firm 2 to always deter antitrust violations, which leads GOV to file suit on its own, as long as its own litigation costs are not too high.

Let us now turn to firm 1’s decision. Consider first the case where firm 1 learns that the type is $RC_1$. Given $\beta_1$, $\beta_2$, and $\sigma_F$, firm 1 randomizes between $A$ and $\neg A$ when

\[ (q + (1 - q)(1 - \sigma_{G2})) \left[ \beta_1(rT - L_1) + (1 - \beta_1)T \right] + (1 - q)\sigma_{G2} (rT - L_1) \]

is zero. Firm 1 chooses $A$ given $RC_1$ when (14) is positive, and $\neg A$ when it is negative.

Next consider the case where firm 1 learns that the type is $IC_2$. In this case, firm 1 randomizes between $A$ and $\neg A$ when
(15) \[ (1-q + q(1-\sigma_{g2}))\left[\beta_1((1-r)T - L_1) + (1-\beta_2)T\right] + q\sigma_{g2}((1-r)T - L_1) \]

is zero. Firm 1 chooses \( A \) given \( IC_2 \) if (15) is positive, and \( \neg A \) if it is negative.

Using the above best response conditions for each of the players, we can derive the sequential equilibria and corresponding welfare values of the game with both private and public enforcement. We measure welfare as the sum of the three players’ equilibrium payoffs. The results are presented in the following proposition.

**Proposition 2.** The subgame perfect Nash equilibria of the game with public and private enforcement.

(A) If \( L_2 < (1-r)T < L_1 < rT \), then the unique subgame perfect equilibrium is \( \alpha^*_1 = 1 \), \( \alpha^*_2 = 0 \), \( \sigma^*_{g2} = 0 \), \( \beta^*_1 = \beta^*_2 = 1 \) and equilibrium welfare is \( W^* = prx - p(L_1 + L_2) \).

(B) If \( L_2 < (1-r)T < rT < L_1 \), then \( \alpha^*_1 = \alpha^*_2 = 0 \) and \( W^* = 0 \).

(C) If \( (1-r)T < L_2 < rT < L_1 \) or \( (1-r)T < L_2 < L_1 < rT \), then \( \alpha^*_1 = 1 \), \( \alpha^*_2 = 0 \), \( \sigma^*_{g2} = 0 \), \( \beta^*_1 = 0 \), \( \beta^*_2 = 1 \), and \( W^* = px \).

(D) If \( L_2 < L_1 < (1-r)T < rT \), then \( \alpha^*_1 = \alpha^*_2 = 1 \), \( \sigma^*_{g2} = 0 \), \( \beta^*_1 = \beta^*_2 = 1 \), and \( W^* = px - (1-r)x - (L_1 + L_2) \).

(E) If \( (1-r)T < rT < L_2 < L_1 \), then \( \beta^*_1 = \beta^*_2 = 0 \) and the equilibrium outcomes are the same as in the game of pure public enforcement (see proposition 1).

If firm 2’s litigation costs are higher than its expected benefit from suing when firm 1 takes an illegal action (region E), then firm 2 never sues regardless of what it learns about the legality of firm 1’s action. Knowing this, the government acts as if there is no chance of private enforcement. In this case, the full game reduces to the game with pure public enforcement, the solution to which was characterized in proposition 1.

If firm 2’s litigation costs are not prohibitively high (regions A through D), then it sues at least when firm 1 takes an illegal action. In this case, GOV never sues (\( \sigma^*_{g2} = 0 \)) since it can count on firm 2 to always sue if firm 1 takes an illegal action. GOV knows
that it is not perfectly informed and might mistakenly sue firm 1 for taking an efficient action. Therefore, it prefers to delegate enforcement to firm 2 whenever it can. GOV only acts if it expects no private enforcement, as is the case if firm 2’s litigation costs are prohibitively high. As long as this is not the case, public enforcement completely gives way to private enforcement when the two are in play.

If firm 2’s litigation costs are lower than its expected benefit from suing when firm 1 takes a legal action (regions A, B, D), it always sues, regardless of the legality of firm 1’s action. Firm 1 takes the action, whether or not it is legal, if its litigation costs are sufficiently low (region D), takes neither type of action if its litigation costs are sufficiently high (A), and takes only a legal action if its costs are intermediate (region B).

If, instead, firm 2’s litigation costs are higher than its expected benefits from wrongfully accusing firm 1, but lower than its expected benefits from rightfully accusing firm 1 (region C), as is the case if the court is sufficiently accurate ($r$ is close enough to 1), then firm 2 sues if and only if firm 1 takes an illegal action. In this case, firm 1 need not worry about being sued if it takes the efficient action, and thus always takes the efficient action. On the other hand, if it takes the illegal action, it expects that it will be sued by firm 2 with certainty, and therefore does not take the illegal action. Hence, firm 1 only takes the action if it is legal, and firm 2 only sues if firm 1 takes an illegal action.

From the results in proposition 2, we can easily deduce the equilibria of the reduced game with pure private enforcement. In regions (A) through (D), GOV never sues, so the results for the full game with private and public enforcement are exactly the same as those for the reduced game with pure private enforcement. This only leaves region (E). But in this region firm 2’s litigation costs are so high that it never sues. So in this region, in the game with pure private enforcement, the equilibrium actions are simply $\alpha_1^* = \alpha_2^* = 1$, $\beta_1^* = \beta_2^* = 0$, and welfare is $W^* = px - (1 - p)x$.

VI. Social Welfare

We have solved the full game with private and public enforcement and the reduced games with pure private and pure public enforcement. We now compare these games in terms of social welfare. For the comparisons, we assume that under pure public
enforcement GOV wants to sue if it knows that firm 1 always takes the illegal action, and firm 1 does not want to take the illegal action if GOV always sues, that is, we focus on region (i) of parameter space (see proposition 1). This implies that under private enforcement firm 1 does not want to take the illegal action if firm 2 always sues either, that is, region (D) of parameter space is also excluded (see proposition 2). We start by analyzing the welfare effects of adding private enforcement to public enforcement.

Social welfare is comprised of various elements, including the probability that an illegal action is deterred, the probability that an illegal action is taken but overturned by the court, the probability that a legal action is deterred, the probability that a legal action is overturned, and the expected costs of trial. Table 1 summarizes the effects of adding private enforcement to public enforcement on these elements of social welfare.

INSERT TABLE 1.

In region (A) of parameter space, firm 1 chooses to take the action if and only if it is legal and firm 2 chooses to sue firm 1 whether firm 1 takes the legal or illegal action. In this case, with private enforcement, the probability that firm 1 is deterred from taking the illegal action is 1, and the probability that an illegal action is overturned by the court is 0, as indicated in the third column of table 1. Therefore the probability of that an illegal action is deterred or overturned is $1 + 0 = 1$.

On the other hand, under pure public enforcement, the probability that an illegal action is deterred is $1 - \gamma_2^*$, and the probability that it is overturned is $\gamma_2^* q \sigma^* r$, which is the probability that it is taken times the probability that GOV receives a signal that it was taken times the probability that GOV sues times the probability that the court’s verdict is correct, as indicated in the fourth column of the table. Therefore, the probability that an illegal action is deterred or overturned is $1 - \gamma_2^* + \gamma_2^* q \sigma^* r$. Since $1 > 1 - \gamma_2^* + \gamma_2^* q \sigma^* r$, an illegal action is more likely to be deterred or overturned with private enforcement than under pure public enforcement, as indicated in the last column of the table.

In region (A), with private enforcement, the probability that a legal action is not deterred is 1, that is, firm 1 always takes the legal action, and the probability that a legal action is not overturned is $1 * 1 * r = r$, which is the probability that a legal action is
taken times the probability that the action is sued times the probability that the court’s 
verdict is correct. Therefore, the probability of a legal action is $1 \ast r = r$. On the other 
hand, under pure public enforcement, the probability that a legal action is not deterred is 
also 1, and the probability that a legal action is not overturned is $1 \ast q + 1 \ast (1 - q)(1 - \sigma^*_G) + 1 \ast (1 - q) \ast \sigma^*_G \ast r = 1 - (1 - q)\sigma^*_G(1 - r)$, which is the probability that a legal 
action is taken but not sued by GOV or taken and sued by GOV but not overturned. Thus, 
the probability of a legal action is $1 \ast 1 - (1 - q)\sigma^*_G(1 - r) = 1 - (1 - q)\sigma^*_G(1 - r)$. Since $r < 1 - (1 - q)\sigma^*_G(1 - r)$, the probability of a legal action is smaller with private 
enforcement than under pure public enforcement.

With private enforcement, the trial probability is the probability $p$ that firm 1 has 
the opportunity to take a legal action. Therefore, the expected costs of trial are $p(L_1 + L_2)$ . 
On the other hand, under pure public enforcement, the trial probability is $\sigma^*_G [p(1 - q)+(1 
-p)q \gamma^*_2$], which is the probability of public suit plus the probability of private suit. Thus, 
the expected costs of trial are $\sigma^*_G [p(1 - q)+(1 - p)q \gamma^*_2]/(L_1 + L_{Gov})$. The expected costs 
of trial are larger with private enforcement than under pure public enforcement in region 
(A) if and only if $p(L_1 + L_2) > \sigma^*_G [p(1 - q)+(1 - p)q \gamma^*_2]/(L_1 + L_{Gov})$.

Similarly, we can compare the elements of social welfare under private and public 
and under pure public enforcement in the other relevant regions of parameter space. With 
private enforcement, the illegal action is deterred with probability 1 in all relevant regions. 
In contrast, under pure public enforcement, the illegal action is never deterred with 
probability 1. The probability of an illegal action being deterred or overturned by the 
court is always higher with private enforcement than under pure public enforcement.

Under pure public enforcement, there is no region in which firm 1 is deterred 
from taking a legal action. That is, firm 1 always takes a legal action, as explained in 
section IV. In contrast, with private enforcement, firm 1 is completely deterred from 
taking a legal action in region (B). In this case, the court is sufficiently inaccurate that 
firm 2 always sues when firm 1 takes a legal action, preferring to take a chance with the 
court than suffer a certain loss in market share. Firm 1 therefore prefers not to take a legal
action, to avoid being wrongfully sued and possibly wrongfully required to pay damages since the court is relatively inaccurate.

The probability of a legal action being taken and not overturned by the court is higher under pure public enforcement than with private enforcement in all regions except (C), where legal actions are always taken but never sued under private and public enforcement, and always taken but occasionally sued and overturned by the court under pure public enforcement. In the majority of cases, legal actions are taken and survive with higher probability under pure public enforcement.

In region (A), where firm 1 only takes the action if it is efficient and firm 2 always sues firm 1, the effect on the expected costs of trial of adding private enforcement to public enforcement is ambiguous. In contrast, in region (B), where firm 1 never takes the action and firm 2 always sues if an action is taken, and in region (C), where firm 1 only takes the action if it is legal and firm 2 only sues if firm 1 takes the illegal action, the probability of trial is 0 with private enforcement, whereas it is greater than 0 under pure public enforcement. Thus, the probability of trial is always higher under pure public enforcement in regions (B) and (C).

In region (C), which obtains if the court is sufficiently accurate \((r\) is sufficiently close to 1), the probability of an illegal action is lower, the probability of a legal action is higher, and the expected costs of trial are lower with private enforcement than under pure public enforcement. Therefore, in this region, private enforcement unambiguously increases social welfare. In fact, private enforcement achieves the overall welfare-maximizing outcome in this region. If the court is sufficiently accurate, the truth about the type of firm 1’s action would likely be known if firm 1 were to take the action and be brought to court for it. Firm 2 then always sues if it learns that firm 1 took the illegal action, and never sues if it learns that firm 1 took the legal action. Firm 1 always takes the legal action and never takes the illegal action. This equilibrium outcome maximizes overall social welfare since no illegal actions are taken and no legal actions are sued.

The following proposition presents the necessary and sufficient conditions for welfare to be higher with private enforcement than under pure public enforcement in the other regions of parameter space.
Proposition 3. The welfare effect of adding private enforcement to public enforcement.

(A) If \( L_2 < (1-r)T < L_1 \), private enforcement increase welfare if and only if
\[
L_{GOV} > x\left(\frac{q(rT+L_3)((1-r)x+(L_2+L_3))-(1-q)(1-r)xT+(1-r)xL_1+L_T}{(rT+L_3)(1-r)x+(1-q)(L_2+L_3)+(1-q)x}\right).
\]

(B) If \( L_2 < (1-r)T < T < L_1 \), private enforcement increase welfare if and only if
\[
L_{GOV} > x(r+q-1-L_T)\left(\frac{1-q}{1-r}\right).
\]

(C) If \((1-r)T < L_2 < T \), private enforcement unambiguously increase welfare.

In regions (A) and (B), where the court is not so accurate that the first-best outcome is achieved with private enforcement, adding private enforcement to public enforcement increases welfare if and only if GOV is sufficiently inefficient (\( L_{GOV} \) is high enough, and \( q \) is low enough). In these regions, firm 2 would sue with positive probability if firm 1 took an efficient action. In region (B), this leads firm 1 to avoid taking an efficient action. In region (A), it does not prevent firm 1 from taking the efficient action, but firm 1’s efficient action is nevertheless overturned sometimes since the court is not very accurate. Society prefers to avoid these inefficiencies by disallowing private suits if GOV has low enough litigation costs and is not too misinformed.

Comparing the efficiency of the three mechanisms, pure private enforcement, pure public enforcement, and public and private enforcement, yields another interesting result, which we state in the following proposition.

Proposition 4. As long as society prefers the outcome under pure public enforcement to the one under no enforcement at all, pure private enforcement is a weakly dominated mechanism, and in particular, it is weakly dominated by the mechanism of private and public enforcement.

In regions (A) through (C), GOV never sues (see proposition 2), so the mechanism of pure private enforcement yields the same welfare as the one of private and public enforcement. Moreover, in these regions, the mechanism of pure public enforcement yields higher welfare than the one of private and public enforcement if GOV is sufficiently efficient (see proposition 3). In region (E), GOV sues with positive
probability, but firm 2 never sues (see proposition 2). Thus, in this region, the mechanism of private and public enforcement yields the same welfare as the mechanism of pure public enforcement, and these mechanisms result in some public enforcement. Moreover, in this region, the mechanism of pure private enforcement results in no enforcement at all. Hence, as long as society prefers some public enforcement to no enforcement at all, pure private enforcement is a weakly dominated mechanism. In particular, it can never yield more, and can yield less, welfare than the mechanism of public and private enforcement.

VII. Optimal Mechanism

From the analysis in the previous section, we know that if the court is sufficiently accurate, a mechanism with private enforcement achieves the first best outcome. But if the court is not so accurate, neither of the mechanisms analyzed so far achieves the first best outcome. What kind of mechanism maximizes social welfare in general?

So far, we have assumed that private damages are simple, in the sense that a winning plaintiff receives exactly the amount of damages, $T$, resulting from an action by the defendant. Private damages can be multiplied so that the plaintiff receives a multiple, say $N \geq 0$, of $T$. We have also assumed that damages are coupled, in the sense that the plaintiff receives what the defendant pays. Private damages can be decoupled so that the plaintiff receives a fraction or multiple, say $\delta \geq 0$, of what the defendant pays. It turns out that the mechanism that maximizes social welfare in general requires private enforcement with damages that are both appropriately multiplied and decoupled.

To prove this, we redefine the mechanisms with private enforcement in terms of the damage multiplier and decoupler. Firm 1 and firm 2’s payoffs are the same as before except when the court rules against the defendant. In this case, firm 1 pays $NT$, and firm 2 receives $\delta NT$. The rest of the payment or subsidy, $(1- \delta)NT$, is assumed to go to, or come from, the public at large. As before, the compensatory damage payment is assumed to be a non-distortionary redistribution between the firms.\(^5\) The regions of parameter space and best response conditions of the mechanisms with private enforcement are easily redefined.

\(^5\) It is possible that excessive compensation result in bankruptcy for the defendant and thus in a change in market structure. We assume that $N$ is not so large as to drive the defendant out of the market.
in terms of $NT$ and $\delta NT$ instead of $T$. Within this more general framework, we derive the following result about the nature of the optimal mechanism.

**Proposition 5.** The socially optimal outcome, in which firm 1 takes the action only if it is legal and firm 2 only sues if firm 1 takes an illegal action, is achieved only through private enforcement with a damage multiplier $N$ and decoupler $\delta$ that satisfy the following two conditions: 

\[ N > \frac{T - \delta L}{r T} \quad \text{and} \quad \frac{L_2}{r T} < \delta N < \frac{L_2}{(1-r) T}. \]

Condition (1) ensures that firm 1 does not take an illegal action if firm 2 would sue firm 1 for taking it. Condition (2) ensures that firm 2 sues firm 1 if firm 1 takes an illegal action, and does not sue firm 1 if it takes a legal action. Together these conditions guarantee that firm 1 is always deterred from taking an illegal action and firm 2 never strategically abuses of the laws, so that firm 1 is never deterred from taking an efficient action. If these two conditions are satisfied, private enforcement yields the overall social optimum. Notice that the two conditions are easier to satisfy the more accurate is the court (the closer is $r$ to 1). Private enforcement can achieve the first best outcome even with simple coupled damages ($N=1$, $\delta=1$) if the court is sufficiently accurate, as we found previously.

In general, however, achieving the first best outcome with private enforcement requires that damages be both multiplied and decoupled. Multiplying without decoupling can ensure that condition (1) is satisfied, but cannot always ensure that (2) is satisfied. Decoupling without multiplying can ensure that (2) is satisfied, but cannot always ensure that (1) is satisfied. For example, suppose $L_1 = 1$ million, $L_2 = 0.5$ million, $T = 5$ million, and $r = 0.7$. In this case, the conditions for overall optimality under private enforcement are (1) $N > 1.14$ and (2) $1/7 < \delta N < 1/3$. With $N=1$ and $\delta=1/5$, condition (2) is satisfied but (1) is not. With $N=3$ and $\delta=1$ (coupled treble damages), condition (1) is satisfied but the second inequality in condition (2) is not.

In this example, private enforcement with coupled treble damages deters firm 1 from taking an illegal action, but does not prevent firm 2 from suing firm 1 for taking a legal action, and thus does not yield the welfare-maximizing outcome. Thus, achieving the social optimum requires decoupling damages as well. With $N=3$ and $\delta=1/10$ (treble
damages decoupled tenfold), both conditions are satisfied and private enforcement yields the socially optimal outcome.

If instead $L_1 = 2$ million, $L_2 = 3$ million, the conditions are $N > 0.57$ and $4/7 < \delta N < 4/3$, which are not satisfied with $N=3$ and $\delta=1$, but are satisfied with $N=3$ and $\delta=1/3$. If $L_1 = 4$ million, $L_2 = 4.5$ million, the conditions are $N > 1.14$ and $8/7 < \delta N < 8/3$, which again are not satisfied with $N=3$ and $\delta=1$, but are satisfied with $N=3$ and $\delta=1/3$. In general, the higher are firm 2’s litigation costs, the less society needs to worry about firm 2 strategically abusing of the laws, and hence the higher is the decoupler required for private enforcement to achieve the social optimum. But as long as the litigation costs of firms are not too high, a system of private enforcement with treble damages can be improved by awarding plaintiffs less than the treble damages paid by defendants.

Notice lastly that for any damage multiplier and decoupler $(N, \delta)$, one can find another multiplier and decoupler $(N', \delta')$ such that $N' > N$ and $N'\delta' = N\delta$: $(N' = mN, \delta' = (1/m)\delta)$ for $m > 1$. With $(N', \delta')$, condition (1) is more easily satisfied, and condition (2) is just as easily satisfied as with $(N, \delta)$. Thus, changing the damage structure from $(N, \delta)$ to $(N', \delta')$ cannot make it harder, and may make it easier, for private enforcement to achieve the social optimum. For example, given a system of private enforcement with simple coupled damages $(N=1, \delta=1)$, society can do no worse and may do better by going to a system of private enforcement with treble damages decoupled threefold $(N=3, \delta=1/3)$. This will reduce the propensity of firms to take an illegal action without changing the propensity of firms to strategically abuse of the antitrust laws.

VIII. Conclusion

In this paper, we developed a strategic model of antitrust crime and law enforcement that yielded several results that may be useful to policy-makers.

First, under a system of coupled damages, like the one currently in operation in the U.S., adding private enforcement to public enforcement is always socially beneficial if the court is sufficiently accurate. Under pure public enforcement, firms are never deterred from taking efficient actions, but with private enforcement, they are sometimes deterred from taking efficient actions unless the court is sufficiently accurate. On the
other hand, under pure public enforcement, firms are never completely deterred from taking illegal actions, but with private enforcement, they are completely deterred from taking illegal actions if the court is sufficiently accurate. Thus, if the court is sufficiently accurate, adding private enforcement to public enforcement is unambiguously socially beneficial. Hence, to the extent that policy-makers trust in the ability of the courts to arrive at the truth, the model suggests that they should permit and encourage private enforcement of the antitrust laws. Second, under a system of coupled damages, if the court is less accurate, adding private enforcement is socially beneficial only if public enforcement is sufficiently inefficient on its own. If the court is less accurate, adding private enforcement causes firms to take fewer efficient actions as their rivals now have sufficient incentive to strategically abuse of the laws, although it causes them to take fewer illegal actions as well. In this case, private enforcement is socially beneficial only if the government is sufficiently inefficient in litigation and sufficiently misinformed, so that firms would often take illegal actions in the absence of private enforcement. Therefore, to the extent that the court is prone to mistake and full-time public enforcers are efficient in litigation because of increasing returns to scale, the model suggests that policy-makers should discourage private enforcement of the antitrust laws in a system with coupled damages.

Third, pure private enforcement always yields weakly lower social welfare than private enforcement combined with public enforcement. Adding public enforcement to private enforcement cannot harm and may benefit society, even though public enforcers may not be as well informed as private enforcers. Public enforcement simply gives way to private enforcement whenever private enforcement is socially preferable. Therefore, the model suggests that policy-makers should always favor the maintenance of a public enforcer of the antitrust laws, such as the Antitrust Division of the US Department of Justice, assuming of course that the public enforcer is not prone to malfeasance.

Fourth, the social welfare maximizing outcome can only be achieved under a system of private enforcement with damages that are both multiplied and decoupled. Multiplying damages ensures that firms do not take illegal actions, while decoupling them ensures that firms do not strategically abuse of the antitrust laws. If the litigation costs of firms are not too high, they have incentive to strategically abuse of the laws. In
this case, if treble damages are already in operation, like in the U.S., then the model suggests that the damages should be decoupled so that plaintiffs receive only a fraction of what defendants pay. Finally, the model suggests that a system with private enforcement and simple coupled damages, like the ones currently in operation in Australia and Canada, can always be improved by multiplying private damages say threefold and decoupling them threefold. This would reduce the incentives of firms to violate the antitrust laws, without affecting the incentives of firms to strategically abuse of the antitrust laws.

The basic model developed in this paper could be extended in at least two interesting ways. First, we assumed that firm 2 chooses whether or not to sue firm 1 after the government has chosen whether or not to do so. In an alternative setup, firm 2 would move before the government. Then, if firm 2 did not sue, the government might infer that firm 1 took a legal action. In other words, private lawsuit would be a public signal. One could explore the additional implications of this type of signaling for the efficiency of private antitrust enforcement. Second, we assumed that firm 2 cannot subsidize the government to sue on its behalf. Private support for public lawsuit is another possible mechanism, which may have the benefit of inducing efficient information revelation and making public enforcement more effective, but the disadvantage of creating a risk of capture. We leave these interesting extensions for future research.

Mathematical Appendix

Proof of Proposition 1.

(i.a) Suppose \( T < (1-q)((1-r)T + L_4) < q(rT + L_4) \) and \( \left( r - \frac{p(1-q)}{p(1-q)+(1-p)q} \right)x > L_{GOV} \). There are two Nash equilibria in this range, but only one is stable. Suppose \( \gamma_2 \in (0,1) \) is an equilibrium. Then (5) is binding and \( \sigma_G = \frac{T}{q(rT+L_4)} \in (0,1) \). Then (4) is positive, so \( \gamma_1^* = 1 \). From (3),

\[
\gamma_2^* = \frac{p(1-q)(1-r)x}{(1-p)q} \frac{L_{GOV}}{r=L_{GOV}} \in (0,1).
\]

Therefore, \( \gamma_1^* = 1, \gamma_2^* = \frac{p(1-q)(1-r)x}{(1-p)q} L_{GOV} \), \( \sigma_G^* = \frac{T}{q(rT+L_4)} \) is a Nash equilibrium. Suppose \( \gamma_2 = 1 \) is part of an equilibrium. From (3), \( \left( r - \frac{p(1-q)}{p(1-q)+(1-p)q} \right)x > L_{GOV} \) implies \( \sigma_G = 1 \) if \( \gamma_2 = 1 \). But from (5), \( T < q(rT+L_4) \) implies \( \gamma_2 = 0 \) if \( \sigma_G = 1 \), which is a contradiction. Thus \( \gamma_2 = 1 \) is not an equilibrium. Suppose \( \gamma_2 = 0 \). From (3), if \( \gamma_1 \neq 0, B \)
= 1, so that $\sigma_G = 0$. Then, from (5), $\gamma_2 = 1$, which is a contradiction. Thus, $\gamma_2 = 0$ is not an equilibrium if $\gamma_1 \neq 0$. But if $\gamma_1 = 0$, then any $\sigma_G \in (0,1]$ is optimal for GOV, and from (4) and (5), $\gamma_1^* = \gamma_2^* = 0$ if $\sigma_G^* \in (\sigma_{Ga},1]$. Since $(1-q)((1-r)T + L_t) < T$, $\sigma_{Ga} < 1$, and thus, $\sigma_G^* \in (\sigma_{Ga},1]$ exists. Therefore, $\gamma_1^* = \gamma_2^* = 0, \sigma_G^* \in (\sigma_{Ga},1]$ is a Nash equilibrium. However, this equilibrium is not stable. Consider small perturbations $\gamma_1 = \varepsilon_1 > 0$ and $\gamma_2 = \varepsilon_2 > 0$ from the equilibrium strategies $\gamma_1^* = \gamma_2^* = 0$. If GOV observes an action and receives the IC$_2$ signal, its belief that the action is of type IC$_2$ is then $\mu = \frac{\varepsilon_1 (1-q) p}{\varepsilon_1 (1-q) p + \varepsilon_2 (1-p) q}$ . Take $\varepsilon_{1i} = \frac{1}{k}, \varepsilon_{2i} = \frac{1}{k'}$. Note that these perturbations satisfy $\varepsilon_1 > \varepsilon_2$, and are thus reasonable since deviating from $\gamma_1 = 0$ is less costly for firm 1 than deviating from $\gamma_2 = 0$. As $k \to \infty$, $\mu \to 1$, so that $(r-\mu)x < L_{GOV}$ and therefore $\sigma_G^* = 0$. So $\gamma_1^* = \gamma_2^* = 0$ is no longer optimal. Hence, $\gamma_1' = \gamma_2' = 0$ is not a stable equilibrium. 

(i.b) Suppose $(1-q)((1-r)T + L_t) < T < q(rT + L_t)$ and $(r-\frac{p(1-q)}{p(1-q)+p(1-r)q})x > L_{GOV}$ . As in (i.a), $\gamma_2 = 1$ is not an equilibrium. Suppose $\gamma_2 = 0$. From (4), $(1-q)((1-r)T + L_t) < T$ implies $\gamma_1 = 1$ even if $\sigma_G = 1$. For $\gamma_1 = 1$ and $\gamma_2 = 0$, (3) is always negative, and thus, $\sigma_G = 0$. Then, from (5), $\gamma_2 = 1$, which is a contradiction. Thus, $\gamma_2 = 0$ is not an equilibrium. Therefore, in equilibrium, $\gamma_2 \in (0,1)$, so that equation (5) is binding and $\sigma_G^* = \frac{T}{q(rT + L_t)} \in (0,1)$. Then (4) is positive, so $\gamma_1' = 1$. From (3), $\gamma_2' = \frac{p(1-q)}{p-q} \frac{L_t}{(rT + L_t)} \in (0,1)$. Therefore, if $T < q(rT + L_t)$ and $(r-\frac{p(1-q)}{p(1-q)+p(1-r)q})x > L_{GOV}$ , the unique stable equilibrium is $\gamma_1^* = 1$, $\gamma_2^* = \frac{p(1-q)}{p-q} \frac{(1-r)x + L_{GOV}}{(r-x)L_{GOV}} \in (0,1)$, $\sigma_G^* = \frac{T}{q(rT + L_t)} \in (0,1)$, and equilibrium welfare is $W^* = px - px \left(\frac{1-q}{q}\right) \left(\frac{(1-r)x + L_{GOV}}{(r-x)L_{GOV}}\right) - px \left(\frac{1-q}{q}\right) \left(\frac{L_t}{(r-x)L_{GOV}}\right)$. 

(ii) Suppose $T > q(rT + L_t)$ and $(r-\frac{p(1-q)}{p(1-q)+p(1-r)q})x > L_{GOV}$ . The condition $T > q(rT + L_t)$ implies $\gamma_2 = 1$ even if $\sigma_G = 1$. Since $T > q(rT + L_t) > (1-q)((1-r)T + L_t)$, $\gamma_1 = \gamma_2 = 1$. Then $(r-\frac{p(1-q)}{p(1-q)+p(1-r)q})x > L_{GOV}$ implies $\sigma_G = 1$. Hence, the unique equilibrium is $\gamma_1^* = \gamma_2^* = 1$ and $\sigma_G^* = 1$, with $W^* = px - p(1-q)(1-r)x - (1-p)(1-q)x - (p(1-q) + (1-p)q)(L_t + L_{GOV})$. 

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(iii) Suppose \( \left( r - \frac{p(t-q)}{p(t-q)+r(t-p)} \right)x < L_{GOV} \). As in (i.a), \( \gamma'_1 = \gamma'_2 = 0 \) and \( \sigma_{GOV}^* = (\sigma_{G2}, 1) \) is an equilibrium but is not stable. Moreover, \( \gamma_1 \neq 0 \) and \( \gamma_2 = 0 \) is not part of an equilibrium either since it implies that \( \sigma_{G2} = 0 \), which implies \( \gamma_2 = 1 \), which is a contradiction. If \( \gamma_2 \in (0,1) \), \( \sigma_{G2} = \frac{T}{q(rT+L)} \in (0,1) \), so that (4) is always positive and \( \gamma_1 = 1 \). But for GOV to randomize it must be \( \left( r - \frac{p(t-q)}{p(t-q)+r(t-p)} \right)x = L_{GOV} \). This contradicts the parameter range since

\[
\left( r - \frac{p(t-q)}{p(t-q)+r(t-p)} \right)x < \left( r - \frac{p(t-q)}{p(t-q)+r(t-p)} \right)x < L_{GOV} .
\]

Thus, the mixed strategies are not an equilibrium. If \( \gamma_2 = 0 \), (5) is positive for \( \sigma_{G2} \), so (4) is positive and \( \gamma_1 = 1 \). For \( \gamma_1 = \gamma_2 = 1 \), the condition \( \left( r - \frac{p(t-q)}{p(t-q)+r(t-p)} \right)x < L_{GOV} \) implies \( \sigma_{G2} = 0 \) when \( \gamma_1 = \gamma_2 = 1 \). Thus the unique stable equilibrium is \( \gamma'_1 = \gamma'_2 = 1 \) and \( \sigma_{GOV}^* = 0 \), with \( W^* = px - (1-p)x \). Q.E.D.

**Proof of Proposition 2.**

(A) Suppose \( L_2 < (1-r)T < L_1 < rT \). Then (14) is always positive, so \( \alpha_1^* = 1 \). From (8), \( \beta_1^* = 1 \) since \( L_2 < (1-r)T \). If \( \alpha_2^* \neq 0 \), then \( \beta_2^* = 1 \) from (9). Knowing this, GOV does not sue, that is, \( \sigma_{G2}^* = 0 \). Thus, from (15), it must be that \( (1-q)((1-r)T-L_1) > 0 \), which yields a contradiction. Therefore, \( \alpha_2^* = 0 \). In this case, from (9), firm 2 randomizes given \( IC_2 \), that is, \( \beta_{G2}^* \in [0,1] \). Since \( \alpha_2^* = 0 \), \( \sigma_{G2}^* = 0 \). From (15), \( \alpha_2^* = 0 \) is optimal for \( \beta_2^* \in (\frac{T}{rT+L_1}, 1] \). But \( \beta_2^* = 1 \) is firm 2’s optimal choice if firm 1 chooses A when \( IC_2 \). Hence, the only subgame perfect equilibrium is \( \alpha_1^* = 1 \), \( \alpha_2^* = 0 \), \( \beta_1^* = \beta_2^* = 1 \), and \( \sigma_{G2}^* = 0 \).

(B) Suppose \( L_2 < (1-r)T < rT < L_1 \). If \( \alpha_1^* \neq 0 \) and \( \alpha_2^* \neq 0 \), \( \beta_1^* = \beta_2^* = 1 \) from (8) and (9). Then, from (7), \( D(-L_{GOV}) + (1-D)(-L_{GOV}) < 0 \), so \( \sigma_{G2}^* = 0 \). Since \( \beta_1^* = \beta_2^* = 1 \) and \( \sigma_{G2}^* = 0 \), from (14) and (15), we obtain \( q(rT-L_1) + (1-q)(rT-L_1) < 0 \) and \( (1-q)((1-r)T-L_1) + q((1-r)T-L_1) < 0 \), which contradicts \( \alpha_1^* \neq 0 \) and \( \alpha_2^* \neq 0 \). If \( \alpha_1^* \neq 0 \) and \( \alpha_2^* = 0 \), \( \beta_1^* = 1 \) from (8). In this case, from (14), for any \( \sigma_{G2}^* \), 

\[
(q + (1-q)(1-\sigma_{G2}^*))(rT-L_1) + (1-q)\sigma_{G2}^*(rT-L_1) < 0 ,
\]

implying that \( \alpha_1^* = 0 \), which is a contradiction. If \( \alpha_1^* = 0 \) and \( \alpha_2^* \neq 0 \), \( \beta_2^* = 1 \) from (9). Thus, from (15), for any \( \sigma_{G2}^* \),
(1 - q + q(1 - \sigma_{G2}))[(1 - r)T - L_1] + q\sigma_{G2}((1 - r)T - L_1) < 0$, so \( \alpha_2^* = 0 \), which is a contradiction.

When \( \frac{r - (1 - q)(1 - \beta_1)\sigma_{G2}}{r T + L_1} < \beta_1 + (1 - \beta_1)(1 - q)\sigma_{G2} \) and \( \frac{r - (1 - q)(1 - \beta_2)\sigma_{G2}}{r T + L_1} < \beta_2 + (1 - \beta_2)q\sigma_{G2} \), (14) and (15) are negative, so \( \alpha_1^* = 0 \) and \( \alpha_2^* = 0 \). Therefore, \( \alpha_1^* = \alpha_2^* = 0 \) and \( \beta_1^* = \beta_2^* = 1 \) is the unique equilibrium.

(C) Suppose \((1 - r)T < L_2 < rT < L_1 \) or \((1 - r)T < L_2 < L_1 < rT \). If \( \alpha_1^* \neq 0 \) and \( \alpha_2^* \neq 0 \), from (8) and (9), \( \beta_1^* = 0 \) and \( \beta_2^* = 1 \). Then, from (12), \( D(-L_{GOV} - (1 - r)x) + (1 - D)(-L_{GOV}) < 0 \), so \( \sigma_{G2}^* = 0 \). Then, from (15), \((1 - q)(1 - r)T - L_1) + q((1 - r)T - L_1) < 0 \), which contradicts \( \alpha_2^* \neq 0 \).

If \( \alpha_1^* = 0 \) and \( \alpha_2^* \neq 0 \), for \((1 - r)T < L_2 < rT < L_1 \), from (9), \( \beta_2^* = 1 \). Then, (15) is negative for any \( \sigma_{G2}^* \), so \( \alpha_2^* = 0 \), which is a contradiction. For \((1 - r)T < L_2 < L_1 < rT \), if \( \alpha_1^* = 0 \) and \( \alpha_2^* \neq 0 \), (14) is positive for any \( \beta_1^* \) and \( \sigma_{G2}^* \), so \( \alpha_2^* \neq 0 \), which is also a contradiction. The equilibrium occurs when \( \alpha_1^* \neq 0 \) and \( \alpha_2^* = 0 \). In this case, from (8), \( \beta_1^* = 0 \). Thus GOV does not sue, that is, \( \sigma_{G2}^* = 0 \). Strategy \( \alpha_2^* = 0 \) is optimal for firm 1 if \( \beta_1^* = 1 \) is firm 2’s optimal choice if firm 1 chooses A when IC2. Hence, the only subgame perfect equilibrium is \( \alpha_1^* = 1, \alpha_2^* = 0, \beta_1^* = 0, \beta_2^* = 1, \) and \( \sigma_{G2}^* = 0 \).

(D) Suppose \( L_2 < L_1 < (1 - r)T < rT \). Then (14) and (15) are always positive for any \( \beta_1, \beta_2, \) and \( \sigma_{G2}^* \), so \( \alpha_1^* = \alpha_2^* = 1 \). Then, from (8) and (9), \( \beta_1^* = \beta_2^* = 1 \). Then, from (12), GOV’s payoff reduces to \( \frac{p(1 - q)}{p(1 - q) + (1 - p)q}(-L_{GOV}) + \frac{(1 - p)q}{p(1 - q) + (1 - p)q}(-L_{GOV}) < 0 \), so \( \sigma_{G2}^* = 0 \).

(E) Suppose \((1 - r)T < rT < L_2 < L_1 \). Then, from (8) and (9), \( \beta_1^* = \beta_2^* = 0 \), so the game reduces to the game of pure public enforcement (see the proof of proposition 1). Q.E.D.

**Proof of Proposition 3.**

(A) If \( L_2 < (1 - r)T < L_1 < rT \), welfare is \( W^* = px - p(L_1 + L_2) \) with private enforcement, and \( W^* = px - px(\frac{1 - q}{q})(\frac{r - L_1}{r - L_2}) + px(\frac{1 - q}{q})(\frac{L_1}{r - L_1}) \) without pure public enforcement.

Welfare with private enforcement is greater if and only if \( L_{GOV} > x(\frac{q(r - L_1)(1 - q)(L_1 + L_2) - (1 - q)(1 - r)(x + (1 - r)x_T + (1 - r)(L_1 + L_2))^T}{(r - L_1)(x + (1 - r)x_T + (1 - r)(L_1 + L_2))}) \).
(B) If \( L_2 < (1-r)T < rT < L_1 \), welfare is \( W^* = 0 \) with private enforcement and 
\[
W^* = px - px\left(\frac{1-q}{q}\left(\frac{(1-r)x + \epsilon}{x - \frac{L_{gov}}{r}}\right)\right) - px\left(\frac{1-q}{q}\left(\frac{\epsilon T}{x - \frac{L_{gov}}{r}(T + L_1)}\right)\right)
\]
with pure public enforcement. Welfare with private enforcement is greater if and only if 
\( L_{gov} > x(r + q - 1 - \frac{(1-q)L_1}{x(r + L_1)}) \).

(C) If \( (1-r)T < L_2 < rT \), welfare is \( W^* = px \) with private enforcement and 
\[
W^* = px - px\left(\frac{1-q}{q}\left(\frac{(1-r)x + \epsilon}{x - \frac{L_{gov}}{r}}\right)\right) - px\left(\frac{1-q}{q}\left(\frac{\epsilon T}{x - \frac{L_{gov}}{r}(T + L_1)}\right)\right)
\]
with pure public enforcement. Welfare with private enforcement is unambiguously greater. Q.E.D.

Proof of Proposition 4.
In regions (A) through (C), with private enforcement, \( \sigma_{G2}^* = 0 \) (proposition 2), so private and public enforcement is welfare-equivalent to pure private enforcement. In regions (A) and (B), private and public enforcement yields lower welfare than pure public enforcement if \( L_{gov} \) is sufficiently low (proposition 3). In region (E), \( \sigma_{G2}^* = \frac{T}{q(r + L_1)} \) and \( \beta_1^* = \beta_2^* = 0 \), so private and public enforcement is welfare-equivalent to pure public enforcement. But in this region, the equilibrium under pure private enforcement is \( \alpha_1^* = \alpha_2^* = 1 \) and \( \beta_1^* = \beta_2^* = 0 \), corresponding to zero enforcement, and equilibrium welfare is \( W^* = px - (1-p)x \). On the other hand, pure public enforcement yields 
\[
W^* = px - px\left(\frac{1-q}{q}\left(\frac{(1-r)x + \epsilon}{x - \frac{L_{gov}}{r}}\right)\right) - px\left(\frac{1-q}{q}\left(\frac{\epsilon T}{x - \frac{L_{gov}}{r}(T + L_1)}\right)\right)
\]
Welfare under pure public enforcement is greater than welfare under pure private enforcement as long as 
\( L_{gov} < \left(\frac{r - p(1-q)}{(1-p)q + p(1-q)}\right)x - \frac{\sigma_{G2}^q(1-q)}{(r + L_1)(1-p)q + p(1-q)}L_1 \), that is, as long as society prefers the outcome under pure public enforcement to the one under no enforcement at all. Q.E.D.

Proof of Proposition 5.
Consider the game with public and private enforcement. If firm 1 takes an action of type \( RC_1 \), firm 2’s best response is to sue if and only if \( (1-r)\delta NT > L_1 \). If firm 1 takes an action of type \( IC_2 \), firm 2 sues iff \( r\delta NT > L_1 \). For any \( \beta_1, \beta_2, \alpha_1, \) and \( \alpha_2, \) GOV sues if and only if 
\[
-D(1-r)\left[x(1-\beta_1) + \beta_1(1-\delta)NT\right] + (1-D)r\left[x(1-\beta_2) - \beta_2(1-\delta)NT\right] - L_{gov} > 0.
\]
For any \( \beta_1, \beta_2, \) and \( \sigma_{G2} \), firm 1 takes an action of type \( RC_1 \) if and only if
\[ (q + (1 - q)σ_{a_2})[T - β_1L - β_1(1 - r)NT] + (1 - q)σ_{a_2} (rT - L_2) > 0. \] Firm 1 takes an action of type \( RC_1 \) if and only if \( (q(1 - σ_{a_2}) + (1 - q))[T - β_1L - β_1rNT] + qσ_{a_2} ((1 - r)T - L_2) > 0. \)

1) Suppose \( L_2 < (1 - r)δNT < rδNT \). Then \( β^*_1 = β^*_2 = 1 \), and GOV’s payoff reduces to \(-D(1 - r)[(1 - δ)NT] + (1 - D)r[-(1 - δ)NT] - L_{gov} < 0, \) so \( σ^*_2 = 0 \). Regardless of firm 1’s action, the outcome is sub-optimal. Even if firm always takes the efficient action and never takes the illegal action, \( α^*_1 = 1 \) and \( α^*_2 = 0 \), the efficient action is still sometimes overturned by the court since firm 2 sues even when firm 1 takes an efficient action.

2) Suppose \( (1 - r)δNT < L_2 < rδNT \). In this range, \( β^*_1 = 0 \) and \( β^*_2 = 1 \), so \( σ^*_{G_2} = 0 \). Therefore, \( α^*_1 = 1 \). Moreover, \( α^*_2 = 0 \) if \( [T - L_1 - rNT] > 0 \). Hence, the socially optimal outcome is attained in this range if \( [T - L_1 - rNT] > 0 \).

3) Suppose \( (1 - r)δNT < rδNT < L_2 \). Then \( β^*_1 = β^*_2 = 0 \), so that the mechanism of private and public enforcement reduces to the mechanism of pure public enforcement, in which the damage multiplier and decoupler play no role. The outcome is sub-optimal in this range since the mechanism of pure public enforcement never yields the socially optimal outcome, \( α^*_1 = 1, α^*_2 = 0 \) (see proposition 1).

Since the outcome under pure public enforcement is sub-optimal, the optimal outcome is attainable only with private enforcement. Moreover, with private enforcement, the optimal outcome is attained only when \( (1 - r)δNT < L_2 < rδNT \) and \( [T - L_1 - rNT] > 0 \).

Hence, the socially optimal outcome is attained only with private enforcement and a damage multiplier and decoupler that satisfy \( N > \frac{T - L_2}{rT} \) and \( \frac{L_2}{1 - r} > δN > \frac{L_2}{rT} \). Q.E.D.

References


Figure 1: The Game Tree With Private and Public Enforcement
Table 1: The Anatomy of Welfare Under Public & Private and Pure Public Enforcement

<table>
<thead>
<tr>
<th>Region</th>
<th>Element of Welfare</th>
<th>Private &amp; Public</th>
<th>Pure Public</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>Illegal action deterred</td>
<td>1 &gt; $1 - \gamma_z'$</td>
<td>$1 &gt; 1 - \gamma_z'^*(1 - q \sigma_e r)$</td>
<td>Private better</td>
</tr>
<tr>
<td></td>
<td>Illegal action overturned</td>
<td>0 &lt; $\gamma_z' q \sigma_e r$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Legal action not deterred</td>
<td>1 = 1</td>
<td>$r &lt; 1 -(1-q) \sigma_e'^*(1-r)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Legal action not overturned</td>
<td>$r &lt; 1 -(1-q) \sigma_e'^*(1-r)$</td>
<td>Public better</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Expected trial costs</td>
<td>$p (L_1 + L_2) &gt;= &lt; \sigma_e' [p(1-q) + (1-p)q \gamma_z'] (L_q + L_{\text{com}})$</td>
<td>Ambiguous</td>
<td></td>
</tr>
<tr>
<td>(B)</td>
<td>Illegal action deterred</td>
<td>1 &gt; $1 - \gamma_z'$</td>
<td>$1 &gt; 1 - \gamma_z'^*(1 - q \sigma_e r)$</td>
<td>Private better</td>
</tr>
<tr>
<td></td>
<td>Illegal action overturned</td>
<td>0 &lt; $\gamma_z' q \sigma_e r$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Legal action not deterred</td>
<td>0 &lt; 1</td>
<td></td>
<td>Public better</td>
</tr>
<tr>
<td></td>
<td>Legal action not overturned</td>
<td>0 &lt; $1 -(1-q) \sigma_e'^*(1-r)$</td>
<td>Private better</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Expected trial costs</td>
<td>$0 &gt; \sigma_e' [p(1-q) + (1-p)q \gamma_z'] (L_q + L_{\text{com}})$</td>
<td></td>
<td>Private better</td>
</tr>
<tr>
<td>(C)</td>
<td>Illegal action deterred</td>
<td>1 &gt; $1 - \gamma_z'$</td>
<td>$1 &gt; 1 - \gamma_z'^*(1 - q \sigma_e r)$</td>
<td>Private better</td>
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<td></td>
<td>Illegal action overturned</td>
<td>0 &lt; $\gamma_z' q \sigma_e r$</td>
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<tr>
<td></td>
<td>Legal action not deterred</td>
<td>1 = 1</td>
<td>$1 &gt; 1 -(1-q) \sigma_e'^*(1-r)$</td>
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<td>Legal action not overturned</td>
<td>1 &gt; $1 -(1-q) \sigma_e'^*(1-r)$</td>
<td>Private better</td>
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</tr>
<tr>
<td></td>
<td>Expected trial costs</td>
<td>$0 &gt; \sigma_e' [p(1-q) + (1-p)q \gamma_z'] (L_q + L_{\text{com}})$</td>
<td>Private better</td>
<td></td>
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