Marginal Subsidies in Tullock Contests

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May 25, 2012

Abstract

In a two-player Tullock contest, we examine a situation where the contest designer has a small amount of resource which can be used either to provide marginal subsidy to a player or to add to the prize directly. Both a direct effect and an indirect effect occur when subsidizing a player. For the direct effect, although either player would increase effort if subsidized, the marginal effect of a subsidy on the recipient's effort is very likely to be greater if applied to the stronger player. The indirect effect is that the weak (strong) player decreases (increases) effort when his opponent is subsidized. The overall effect depends on the accuracy level of the contest and the extent of the ability difference. We show that to maximize the total effort, subsidizing the weak (strong) player is preferred when the contest is sufficiently accurate (inaccurate). When the ability difference becomes larger (smaller), subsidizing the weak (strong) player is more likely to be preferred. Moreover, adding to the prize can be more beneficial than the preferred-subsidy-scheme only when the contest is sufficiently accurate and the ability difference sufficiently small. Our findings extend to an N-player lottery contest.

Key words: Tullock contest, resource, marginal subsidy, add prize.

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1 Introduction

A contest is a situation in which players compete against each other by making irreversible effort, often for a prize or multiple prizes. In practice, setting prizes has been considered as the most important and effective way to attract potential contestants and stimulate competition between participants. As a result, prizes allocations have been studied extensively in the literature of contests.[1]

Besides awarding prizes, subsidizing contestants, which can also motivate contestants to exert effort, has not drawn much attention in the literature. Actually, in the real world, giving subsidies is not rare in contests or "contest" situations.[2] By estimating an econometric model using contractor-level data, Lichtenberg (1990) shows that the US Department of Defense (DoD) encourages private military R&D investment not only by establishing prizes, but also by subsidizing expenditures (costs of making effort) dedicated towards winning the prize.[3] Moreover, he find that: On the surface, it appears that the *marginal subsidy* on the R&D investment (which reduces the recipient's marginal costs of making effort) is zero, but this is only true in the short term. Due to the DoD's policy of allowable-cost determination, the long-run marginal subsidies are substantial.

Marginal subsidies have been used in other types of contests as well. For instance, in a class where students make effort to achieve better academic performance (such as better diploma/degree classifications), it is common for a teacher to offer *marginal help* to a student or a specific group of students. Specifically, marginal help in this case means that the teacher will offer more help when the student exerts more effort. A teacher may run a helpdesk to help students who have questions about her course. A student who does little homework (i.e., he makes little effort) is very likely to gain little from the helpdesk (i.e., he gets little help);[4] while a well-prepared student (i.e., he makes a large effort) may benefit a lot from it (i.e., he receives great help).

The contest organizer, e.g. the DoD or the teacher, may face a budget constraint on the resources that can be used as subsidies. For example, the DoD may

[1] For instance, Clark and Riis (1996, 1998), Moduvanu and Sela (2001), Szymanski and Valletti (2005), Fu and Lu (2009), Akerlof and Holden (2010), Schweinzer and Segev (2011) and among others have been studied the prize allocation within different settings.

[2] The "contest" situations refer to many types of interaction which have been studied in the field of contest theory, such as sports, rent-seeking, litigation, beauty contests, patent races, research and development (R&D), political competition, arm races and war, etc.

[3] Lichtenberg (1988) show that the DoD has conventionally sponsored numerous design competitions to stimulate private investment in defense technology. Nowadays, R&D contests sponsored by the DoD are still very common. For instance, in 2007, the DoD set a prize of 1 million dollars to lessen the weight of more than 20 pounds of batteries a solider carries on a typical four-day mission.

[4] The students who make little effort on studying may even not go to the helpdesk at all.

have a fixed amount of money on providing subsidies and the teacher may have a fixed amount of time for tutoring her students. Then the problem for the contest organizer will be the following: In order to maximize the total effort, how to use the limited resource most efficiently, i.e., which contestant(s) should be subsidized. For instance, the DoD, who aims to encourage private military R&D in some certain field, has to decide which contractor she wants to subsidize, the "underdog" (the weak one) or the "favorite" (the strong one); the teacher, who wants to improve the overall academic performance of her students, may have to decide whether the helpdesk is mainly for helping the less able or more able students.[5] Moreover, can adding the resource directly to the prize (if possible) be more efficient than providing a subsidy? For example, should the DoD use the money to subsidize a contractor or add the money directly to the prize? This paper is an attempt to answer the above questions.

Our analysis is in a Tullock contest. Notice that parameter r in a Tullock Contest Success Function (CSF), which is often referred to as the discriminatory power in the literature, can also be interpreted as the accuracy level of the contest.[6],[7] In this paper, we look at a situation where the contest organizer has a (sufficiently) small amount of resource which can be used to give a marginal subsidy to either player or to add to the prize directly. Intuitively, both a direct effect and an indirect effect occur when subsidizing a player. For the direct effect, though either player would increase effort if subsidized, the marginal effect of the subsidy on the recipient's effort is very likely to be greater if applied to the stronger player. The indirect effect is that the weak player decreases effort when his opponent is subsidized, while the strong player increases effort when his opponent is subsidized. W e show that the overall effect depends on the accuracy level of the contest and the extent of the ability difference: In order to maximize the total effort, subsidizing the strong player is preferred when the contest is sufficiently inaccurate (when $r < r^*$)[8], while subsidizing the weak player is preferred when the contest is sufficiently accurate (when $r > r^*$). When the ability difference becomes larger, subsidizing the weak player is more likely to be preferred (r^* decreases with the

[5] The teacher should decide which group she wants to help because helping the less able students means mostly reviewing the basics while helping the more able students focuses on expanding the basics and making advanced analysis.

[6] Clark and Riis (1996) show that the Tullock CSF can be interpreted as the outcome of a model in which each player's effort is evaluated with an error where the variance of the error is reflected in the parameter r. This justifies the interpretation of r as accuracy of the contest. In particular, Wang (2010) and Fu, Jiao and Lu (2011) specifically interpret r as the contest's accuracy level.

[7] For example, in a gymnastic balance beam competition, if contestants are allowed to perform on the beam multiple times rather than only once (the final mark is the average mark), the result of the contest will be more accurate on average, correspondingly, r will be greater.

[8] See Proposition 2 for details.

ability difference). Moreover, adding to the prize can be more beneficial than the preferred-subsidy-scheme *only* when the players' abilities are sufficiently close and the contest is sufficiently accurate (when $r > 1$). We confirm our findings in a N-player lottery contest (i.e., Tullock contest with $r = 1$): It is strictly beneficial to subsidize the weakest player among the players who are active in the contest.

In our model, providing a marginal subsidy to a player requires that the contest organizer is able to verify the recipient's effort level (or the effort level can be inferred by the contest organizer). For example, the DoD's audits can verify how much investment a contractor has been made on this research project; the teacher can easily infer how much effort a student has made by observing his performance at class and checking his homework. In cases where effort can be fully verified, theoretically, it is easy for the contest organizer to construct a simple contract that implements the maximal individually rational effort and extracts all surplus from the player. Thus, a contest may become suboptimal. We point out that in the real world there are many cases where the contest organizer can verify players' effort levels, but the outcome of the contest is determined by the contest rule itself rather than the contest organizer. For instance, in the R&D research contest sponsored by the DoD, who winning the contest depends on the quality of the research project, which is determined not only by the firms' effort levels (which can be inferred from their investment levels), but also by some random factors.[9] In other words, although the DoD is fully aware of that the strong firm makes more effort than that of the weak firm, the weak firm can still win the contest by working out a higher quality research project, which might only because the weak frim chooses a good research direction by luck. In another example, students' diploma/degree classifications are determined by their exam grades and a contract between the teacher and any of her student is of course not feasible.[10] For example, most universities in the UK award a class of degree (First Class, Second Class, etc) based on student's average exam results.

[9] Clark and Riis (1996) show that the Tullock CSF can be interpreted as the outcome of a noisy performance ranking model in which each player's output consists of an effort term and a noise term. Fu and Lu (2011) generalize Clark and Riis (1996)'s model to a multi-prize case.

[10] In the literature of contract theory, there are also many cases where binding contracts are not possible, and instead contracts have to be incomplete (started by Williamson, 1975, Grossman and Hart, 1986, and Hart and Moore, 1990). For example, although there is no asymmetric information between firm and worker (in our case, between the contest designer and contestant), the outside court can not observe what the firm and worker observe, which implies a binding contracts is difficult to write.

2 Related Literature

Contests literature has greatly expanded since Tullock's seminal work (1980).[11] However, there are only a few papers studying contests with subsidies (or reimbursements) in the existing literature on imperfectly discriminating contests. The following two papers consider lottery contests ($r = 1$) with full reimbursement. Cohen and Sela (2005) show that in a two-player case, if the contest organizer reimburses the winner's cost of effort, there is a unique internal equilibrium where the weak player wins with higher probability than the strong one. Matros (2008) analyzes the N-player model and discusses the properties of all pure-strategy equilibria. In addition, Matros and Armanios (2009)[12] consider reimbursements in a general Tullock's contest with homogeneous players and find that the winner-reimbursed-contest maximizes net total spending while the loser-reimbursed-contest minimizes it. Our settings are different from the above research works mainly in three ways. First, we consider a Tullock contest with heterogenous players. Second, in our model a player is subsidized regardless of who wins eventually, i.e., subsidy is not contingent. Third, we assume there is a fixed small amount of resource which can be used as subsidies and we compare the efficiencies of subsidizing different players.

Actually, contests with reimbursements have also been studied in the auction literature (perfectly discriminating contests). Riley and Samuelson (1981) introduce the Sad Loser Auction where the winner gets his bid back and wins the prize. Goeree and Offerman (2004) analyze the Amsterdam auction in which the highest losing bidder obtains a premium which depends on his own bid. Clark and Riis (2000) consider a situation where a corrupt official who obtains bribes (effort) from firms (players), and allocates a government contract (prize) in an all-pay auction between two firms. They show that an income (effort) maximizing official will favor the firm who is more likely to value the prize less. Moreover, Che and Gale (2003) show that imposing a bidding cap, which handicaps the strong player, better incentivizes both player. Kirkegaard (2010) examines a contest in which a player may suffer from a handicap (which reduces the player's effort by a fixed percentage) or benefit from a head start (which is an additive bonus), and shows that it is generally profitable to give the weak player a head start.

Therefore, the conventional wisdom from the existing auction literature (e.g., Clark and Riis, 2000, Che and Gale, 2003, etc) states that the underdog should be subsidized in order to improve the competitive balance, which increases the effort level. However, our paper suggests that when the contest is sufficiently inaccurate

[11] For a general survey of contests, see Nitzan (1994), Szymanski (2003), Konrad (2007), Corchon (2007) and Congleton et al (2008).

[12] Before their work, Kaplan et al (2002) provide two examples of contests with reimbursements in politics and economics.

(when $r < r^*$), the favorite (strong player) should be subsidized. Actually, when $r \to +\infty$, the Tullock contest becomes an all-pay auction. From this perspective, the Tullock contest will become closer to an all-pay auction as r increases. Indeed, in this paper we find that when the contest is sufficiently accurate (when $r > r^*$), subsidizing the underdog is always preferred, which is consistent with the conventinal wisdom.

The paper that is most closely related to the present paper is Fu, Lu and Lu (2011). They study the optimal design of R&D contests where the contest organizer can split his budget between a prize and efficiency-enhancing subsidies to the firms (players). They adopt a framework where the quality of a firm's product is randomly drawn from a certain distribution which is influenced by both the firm's research capacity and labor input (first proposed by Tan, 1992). Despite distinctive technical differences between our two models, the insights of our analysis, to a large extent, are consistent. They find that in the optimally designed contest, the sponsor may favor the strong firm when the innovation process involves a substantial difficulty or uncertainty. Analogously, we find that when $r < r^*$, i.e., when the contest is sufficiently inaccurate, subsidizing the strong player is preferred to subsidizing the weak player.

3 The Two-player Model

3.1 The Model

There are two risk-neutral players involved in a contest with a single prize V. Each player, say player i, $(i = 1, 2)$, has a linear cost function, $c_i(e_i) = c_i \times e_i$, where e_i refers to player i's effort level and c_i is player i's marginal cost of making effort. Denote $c = c_2/c_1$, assume player 1 is more able than player 2, i.e., $c > 1$. The probability of winning is determined by the following Tullock CSF: In a contest with n contestants, an arbitrary player i wins the prize with probability

$$P_i(e_i, \mathbf{e}_{-i}) = \begin{cases} \dfrac{e_i^r}{\sum_{j=1}^n e_j^r} & \text{if } max\{e_1, ...e_n\} > 0; \\ 1/n & \text{if } max\{e_1, ...e_n\} > 0, \end{cases} \tag{1}$$

where e_i refers to player i's effort level and $\mathbf{e}_{-i} = (e_1, e_2, ...e_{i-1}, e_{i+1}, ..., e_n)$ represents all other $n-1$ players' effort entries.[13] As previously mentioned, the parameter r in (1), which is often referred to as the discriminatory power, can also be interpreted as the accuracy level of the contest. All the parameters, i.e., r, c_1 and c_2, are common knowledge. Each player maximizes his expected utility π_i where

[13] Note that in our two-player model, $\mathbf{e}_{-i} = e_j$ where $j \neq i$.

$\pi_i = P_i(e_i, \mathbf{e}_{-i})V - c_i e_i$. Assume that the effort-maximizing contest organizer has a small amount of resource s which can be used to subsidize either player or simply add to the prize.

Nti (1999) analyzes a model with heterogeneous valuations ($V_1 \geq V_2$) and homogenous abilities ($c_1 = c_2 = 1$). With linear cost functions, there is a close one-to-one relationship between differences in valuations and differences in abilities (i.e., marginal costs). Due to the technical equivalence between heterogeneous valuations and heterogeneous abilities, Wang (2010) obtain the following results from Nti (1999) with simple transformations, which we restate as follows:

Lemma 1 *Without subsidies, there always exists a unique pure-strategy Nash equilibrium for any $r \in (0, \overline{r}]$, where $\overline{r}$ satisfies $c^{\overline{r}} = 1/(\overline{r} - 1)$, it can be shown that $\overline{r}$ decreases from 2 to 1 as c increases from 1 to $+\infty$.*[14] *The equilibrium effort levels are:*

$$e_1 = \frac{c_1^r c_2^r r V}{c_1(c_1^r + c_2^r)^2}, \; e_2 = \frac{c_1^r c_2^r r V}{c_2(c_1^r + c_2^r)^2}, \; TE = \frac{(c_1 + c_2)c_1^{r-1}c_2^{r-1} r V}{(c_1^r + c_2^r)^2}, \tag{2}$$

where $TE = e_1 + e_2$, which stands for total effort.

Proof. See Appendix.[15] ■

Giving a lump-sum subsidy s to either player will not change their equilibrium efforts at all. This is because in equilibium each player's marginal revenue must be equal to his marginal cost, thus, a lump-sum subsidy, which does not change the receipt's marginal cost, will have no effect on players' effort levels in equilibrium. In this paper, we focus on the case that when subsidizing a player, a certain proportion of his cost will be covered by the contest organizer regardless of who wins the contest. This form of subsidy is referred to as marginal subsidy in this paper.

To use s more efficiently, we are going to compare the efficiencies of subsidizing each of the two players. Suppose when the strong player is subsidized, his marginal cost decreases from c_1 to c_1' (from his own perspective) and he makes an effort e_1' in the new equilibrium. Since the total amount of subsidies must be equal to the resource that the contest organizer acquires, we have

$$s = (c_1 - c_1')e_1', \text{ where } e_1' = \frac{c_1'^r c_2^r r V}{c_1'(c_1'^r + c_2^r)^2}. \tag{3}$$

Alternatively, the contest organizer may consider using s to subsidize the weak player. With the marginal subsidy, the weak player's marginal cost of making

[14] Note that the condition $r \in (0, \overline{r}]$ ensures the existence of pure strategy equilibrium for any $c > 1$.

[15] Although the results are analogous to Nti (1999), we provide a proof in Appendix for completeness of our analysis.

effort decreases from c_2 to c_2' (from his own perspective), similarly,

$$s = (c_2 - c_2')e_2', \text{ where } e_2' = \frac{c_1^r c_2'^r rV}{c_1(c_1^r + c_2'^r)^2}. \tag{4}$$

From (3) and (4), we write

$$\frac{c_1' - c_1}{s} = -(e_1')^{-1};$$
$$\frac{c_2' - c_2}{s} = -(e_2')^{-1}.$$

Therefore, we derive

$$\frac{dc_1}{ds} = \lim_{s\to 0} \frac{c_1' - c_1}{s} = \lim_{s\to 0} -(e_1')^{-1} = -(e_1)^{-1}; \tag{5}$$
$$\frac{dc_2}{ds} = \lim_{s\to 0} \frac{c_2' - c_2}{s} = \lim_{s\to 0} -(e_2')^{-1} = -(e_2)^{-1}. \tag{6}$$

Then we have

$$\frac{dc_1}{ds} / \frac{dc_2}{ds} = \frac{e_2}{e_1} = \frac{c_1}{c_2}. \tag{7}$$

From the expression of TE in (2), we derive

$$\frac{dTE^S}{dc_1} = \frac{c_1^{r-2}c_2^{r-1}\{c_2^r[c_2(r-1) + c_1 r] - c_1^r[c_2 + (c_1 + c_2)r]\}rV}{(c_1^r + c_2^r)^3}, \tag{8}$$

$$\frac{dTE^W}{dc_2} = \frac{c_1^{r-1}c_2^{r-2}\{c_1^r[c_1(r-1) + c_2 r] - c_2^r[c_1 + (c_1 + c_2)r]\}rV}{(c_1^r + c_2^r)^3}, \tag{9}$$

where TE^S and TE^W denote the total effort when subsidizing the strong player and the weak player respectively. To compare the efficiencies in increasing total effort by subsidizing the two players respectively, we write

$$\frac{dTE^S}{ds} / \frac{dTE^W}{ds} = (\frac{dTE^S}{dc_1})(\frac{dc_1}{ds}) / (\frac{dTE^W}{dc_2})(\frac{dc_2}{ds}) = (\frac{dTE^S}{dc_1} / \frac{dTE^W}{dc_2})(\frac{dc_1}{ds} / \frac{dc_2}{ds}). \tag{10}$$

Substituting (7), (8) and (9) into (10), we derive[16]

$$\frac{dTE^S}{ds} / \frac{dTE^W}{ds} = \frac{[c + (1+c)r] - c^r[c(r-1) + r]}{c^r[1 + (1+c)r] - [(r-1) + cr]}. \tag{11}$$

[16] Recall that $c = c_2/c_1$.

Condition 1 *The resource s is suffciently small which ensures that:*

$$\begin{aligned} TE^{S\prime} >,< TE^{W\prime} \text{ when } \frac{dTE^S}{ds} >,< \frac{dTE^W}{ds}; \\ TE^{S\prime} >,< TE^{P\prime} \text{ when } \frac{dTE^S}{ds} >,< \frac{dTE^P}{ds}; \\ TE^{W\prime} >,< TE^{P\prime} \text{ when } \frac{dTE^W}{ds} >,< \frac{dTE^P}{ds}, \end{aligned} \tag{12}$$

where $TE^{W\prime}$, $TE^{S\prime}$ and $TE^{P\prime}$ refer to total effort levels after using s to subsidize the weak player, strong player and add to the prize, respectively.[17]

By analyzing (11), we obtain the following results.

Proposition 1 *Given s being sufficiently small which satisfies Condition 1, in order to maximize total effort, it is more efficient to subsidize the weak player when $r > r^*$ and subsidize the strong player when $0 < r < r^*$ since*

$$\frac{dTE^S}{ds} <,> \frac{dTE^W}{ds} \text{ when } r >,< r^*,$$

where r^* satisfies

$$c^{r^*} - \frac{2r^*(c+1)+(c-1)}{2r^*(c+1)-(c-1)} = 0. \tag{13}$$

It can be shown that r^* decreases when c increases and $r^* \in (0.5,\ 0.708)$ when $c \in (1,\ +\infty)$.

Proof. See Appendix. ■

An alternative way of increasing total effort is to add the resource s directly to the prize. If the contest organizer adds s to the prize, using (2) we have

$$\frac{dTE^P}{ds} = \frac{(c_1+c_2)c_1^{r-1}c_2^{r-1}r}{(c_1^r+c_2^r)^2}, \tag{14}$$

where TE^P refers to the total effort when adding s to the prize. If the contest organizer subsidizes the weak player, using (6) we have

$$\frac{dTE^W}{ds} = \frac{dTE^W}{dc_2}(\frac{dc_2}{ds}) = \frac{dTE^W}{dc_2}(-\frac{1}{e_2}). \tag{15}$$

[17] When s is sufficiently small, (12) will hold since $TE^{X\prime} \approx TE + s\frac{dTE^X}{ds}$ where $X = S, W, P$.

Substituting the expression of e_2 in (2) and (9) into (15),

$$\frac{dTE^W}{ds} = \frac{c_2^r[c_1 + (c_1 + c_2)r] - c_1^r[c_1(r-1) + c_2 r]}{c_1 c_2 (c_1^r + c_2^r)}. \tag{16}$$

Using (14) and (16), we further derive

$$\frac{dTE^W}{ds} / \frac{dTE^P}{ds} = [1 + \frac{c^r + 1 - (1+c)r}{c^r(1+c)r}](1 + c^r). \tag{17}$$

Moreover, to compare the effect of adding s to the prize with that of subsidizing the strong player, we write

$$\frac{dTE^S}{ds} / \frac{dTE^P}{ds} = (\frac{dTE^S}{ds} / \frac{dTE^W}{ds})(\frac{dTE^W}{ds} / \frac{dTE^P}{ds}). \tag{18}$$

Substituting (11) and (17) into (18), we derive

$$\frac{dTE^S}{ds} / \frac{dTE^P}{ds} = \{1 + \frac{c - c^r[r - c(1-r)]}{(1+c)r}\}(1 + c^{-r}). \tag{19}$$

By analyzing (17) and (19), we obtain the following results.

Proposition 2 *Given s being sufficiently small which satisfies Condition 1, in order to maximize the total effort, subsidizing the strong player is more efficient than adding to the prize when $0 < r \leq 1$; when $r > 1$, as long as $c \geq 3.6$, subsidizing the weak player is more efficient than adding to the prize.*[18]

Proof. See Appendix. ■

By comparing the three options of using the resources s, i.e., subsidizing the weak player, subsidizing the strong player and add to the prize, it can be concluded that: To maximize the total effort, when $0 < r < r^*$, subsidizing the strong player dominates (the other two options); while when $r^* < r \leqslant 1$, subsidizing the weak player dominates; when $r > 1$ and $c \geq 3.6$, it can be guaranteed that subsidizing the weak player dominates, when $r > 1$ and $c < 3.6$, either subsidizing the weak player or adding to the prize dominates.

[18] We want to emphesis here that when $c < 3.6$ and $r > 1$, it is still possible that subsidizing the weak player dominates adding to the prize. In other words, even with $c^r + 1 - (1+c)r < 0$, (17) can still be greater than unity.

3.2 Discussions

From (2), it can be derived that:

$$\frac{de_1}{dc_1} < 0, \ \frac{de_2}{dc_2} < 0, \tag{20}$$

$$\frac{de_1}{dc_2} < 0, \ \frac{de_2}{dc_1} > 0. \tag{21}$$

Intuitively, both a direct effect and an indirect effect occur when subsidizing a player. Firstly, we discuss the direct effect. Inequalities (20) indicate that subsidizing either player will induce him to make more effort himself. But the marginal effect of a subsidy on the recipient's effort is very likely to be greater if applied to the stronger player. This is because the strong player is more effective in making effort than the weak player. In other words, by bearing the same amount of cost, the strong player can make more effort. Inequalities (21) show the indirect effect. The weak player reduces effort when the strong player is subsidized; in contrast, the strong player exerts more effort when the weak player is subsidized. It is clear that the direct effect favors subsidizing the strong player and the indirect effect favors subsidizing the weak player.

When r is low (i.e., $0 < r < r^*$), the contest involves a lot of noise, both players make relatively small effort in the initial equilibrium with no subsidy, the marginal effect of a subsidy on the recipient's effort is much greater if applied to the strong player. Though subsidizing the strong player will cause the weak player to exert less effort, the overall effect of subsidizing the strong player is greater than that of subsidizing the weak player. Thus, the strong player should be subsidized.

When r is sufficiently high (i.e., $r > r^*$), the conetst becomes more accurate, both players have made relatively large effort in the initial equilibrium without subsidies. The marginal effect of a subsidy on the recipient's effort if applied to the strong player becomes smaller compared to the previous case with $r < r^*$. While subsidizing the weak player, which also induces the strong player to invest more effort, is more effective in increasing the total effort, i.e., the overall effect of subsidizing the weak player is greater. Thus, the weak player should be subsidized.

Moreover, r^* decreasing with c indicates that when the ability difference becomes larger, subsidizing the weak player is more likely to be preferred. This is because when c increases, the weak player becomes weaker relative to the strong player, then subsidizing the weak player will have a greater effect on motivating both players to make more effort. Therefore, the overall effect of subsidizing the weak player is more likely to be greater than that of subsidizing the strong player.

Notice that, our analysis relies on an assumption that the resource s (which can be used as subsidies) is sufficiently small. When the resource s is large, it can not be guaranteed that our results will still hold. However, our analysis can still provide

good insights on which player should be subsidized when tne resource is large since the basic trade-off between the direct effect and indirect effect always occurs when a subsidy is implemented. The assmption that "rescource being sufficiently small" is made for ease of our analysis, otherwise we could not derive a general result on the expression of r^*.[19]

4 The N-player Model

In this section, we consider a model with n risk-neutral contestants in a contest with a single prize V, where the CSF is the Tullock CSF with $r = 1$.[20] We assume an arbitrary player i's marginal cost of making effort e_i is c_i. The contestants are asymmetric in the sense that $0 < c_1 \leq c_2 \leq ... \leq c_n$. We also assume $c_1 < c_n$ in order to rule out the case where $c_1 = c_2 = ... = c_n$. It is straightforward to see that player 1 (whose marginal cost is c_1) is the most able player and player n (whose marginal cost is c_n) is the least able one. Assume that we start with an equilibrium with no subsidy and all players participate in the contest with strictly positive effort levels.[21]

Player i's expected profit is $\pi_i = P_i(e_i, \mathbf{e}_{-i})V - c_i e_i$, where $P_i(e_i, \mathbf{e}_{-i})$ is given by (1) with $r = 1$. The first order condition is

$$\frac{d\pi_i}{de_i} = \frac{V\sum_{j\neq i} e_j}{(e_i + \sum_{j\neq i} e_j)^2} - c_i = 0. \tag{22}$$

It can be verified that the second-order condition for $d^2\pi_i/de_i^2$ is always satisfied as long as $n \geq 2$.

Lemma 2 *(a)*[22] *In the n-player model, the total effort is*

$$TE = \sum_{i=1}^{n} e_i = \frac{V(n-1)}{\sum_{i=1}^{n} c_i}; \tag{23}$$

(b) For any two players with $c_p < c_q$, we show $e_p > e_q$, i.e., the more able the player is, the more effort he makes in equilibrium.

[19] Notice that even with this assumption, we are unable not obtain a closed-form solution of r^* by solving (13).

[20] The general form of the Tullock CSF is known to be rather intractable in all but two-player contests when contestants are asymmetric in their abilities.

[21] This condition requires that for the weakest player (i.e., player n), his expected utility is greater than or equal to zero in equilibrium. We can look at this in a way assuming there are m ($\geq n$) potential players in total and only n players enter the contest actively by exerting strictly positive effort.

[22] Similar results have been derived in Stein (2002) and Ritz (2008). The uniqueness of equilibrium in asymmetric contests is established by Matros (2006).

Proof. See Appendix. ■

As in the two-player model, suppose the contest organizer has a small resource s which can be used to subsidize any player or add to the prize directly.For any two arbitrary players p and q with $c_p < c_q$, initially in the equilibrium without subsidies, their effort levels are $e_q < e_p$. When player p is subsidized, he makes a new effort level e'_p. From player p's perspective, his marginal cost decreases from c_p to c'_p. Also, the total amount of subsidies must be equal to the resource that the contest organizer acquires, i.e.,

$$s = (c_p - c'_p)e'_p = \Delta c_p e'_p.$$

When player q is subsidized, analogously we derive

$$s = (c_q - c'_q)e'_q = \Delta c_q e'_q.$$

Thus, $\Delta c_p e'_p = \Delta c_q e'_q$.

Condition 2 *Assume s is sufficiently small that ensures $e'_q < e'_p$, i.e., a less able player's effort level is still lower than a more able player's effort level after being subsidized respectively.*

It follows that $\Delta c_p < \Delta c_q$. From (23), we can see that in equilibrium the total effort only depends on the sum of the marginal costs, i.e., $\sum_{i=1}^{n} c_i$. Therefore, regardless of whom being subsidized, the contest organizer only cares about the change of the recepient's marginal cost. Thus, subsidizing the less able player (player q) yields a larger total effort than subsidizing the more able player (player p) as $\Delta c_p < \Delta c_q$. It can be further inferred that it is optimal to subsidize the weakest player to maximize the total effort.

If the contest organizer adds s to the prize in place of subsidizing the weak player, from (23) we have

$$\frac{dTE^P}{ds} = \frac{dTE}{dV} = \frac{n-1}{\sum_{i=1}^{n} c_i}, \tag{24}$$

If the contest organizer subsidizes the weakest player, i.e., player n, then

$$\frac{dTE^W}{ds} = \frac{dTE^W}{dc_n}\frac{dc_n}{ds} = \frac{dTE^W}{dc_n}(-\frac{1}{e_n}). \tag{25}$$

From (23), we derive

$$\frac{dTE^W}{dc_n} = -\frac{n-1}{(\sum_{i=1}^{n} c_i)^2}. \tag{26}$$

Because player n is the weakest player who makes the smallest effort (among the players who are willing to enter the contest actively),

$$e_n \leq \frac{TE}{n} = \frac{n-1}{n\sum_{i=1}^{n} c_i}. \tag{27}$$

Substituting (26) and (27) into (25), we derive

$$\frac{dTE^W}{ds} \geq \frac{n}{\sum_{i=1}^{n} c_i} > \frac{n-1}{\sum_{i=1}^{n} c_i},$$

i.e.,

$$\frac{dTE^W}{ds} > \frac{dTE^P}{ds}.$$

Thus, subsidizing the weakest player is more effective in increasing total effort than adding to the prize. We summarize the findings into the following proposition.

Proposition 3 *Given s being sufficiently small which satisfies Condition 2, in order to maximize the total effort, subsidizing the weakest player who is willing to make strictly positive effort is more efficient than subsidizing any other players or adding the resource to the prize.*

The above result is in line with our previous finding in a general two-player model: When $r = 1$, subsidizing the weak player is more effective in increasing total effort than subsidizing the strong player or adding to the prize directly.

5 Concluding Comments

In a two-player Tullock contest, we examine a situation where the contest organizer has a sufficiently small amount of resource which can be used either to provide a marginal subsidy to a player or to add to the prize directly. We show that, to maximize the total effort, subsidizing the strong (weak) player is preferred when the contest is sufficiently inaccurate (accurate). Adding to the prize can be more beneficial than the preferred-subsidy-scheme only when the contest is sufficiently accurate and the players' abilities are sufficiently close.

Analytical tractability has limited most of our analysis to a standard two-player Tullock contest model. However, we show that our main findings continue to hold in a multi-player lottery contest ($r = 1$): To maximize the total effort, subsidizing the weakest player is more efficient than subsidizing other players or adding the resource to the prize.

Both a direct effect and an indirect effect occur when subsidizing a player. For the direct effect, although either player would increase effort if subsidized, the

marginal effect of a subsidy on the recipient's effort is very likely to be greater if applied to the stronger player. The indirect effect is that the weak player decreases effort when the strong player is subsidized, while the strong player increases effort when the weak player is subsidized. Therefore, the direct effect favors subsidizing the strong player and the indirect effect favors subsidizing the weak player.

As previously mentioned, although most of our results are derived under the assumption that the resource is sufficiently small, our analysis can still provide good insights on which player should be subsidized when the resource is large since the basic trade-off between the direct effect and indirect effect always occurs when a subsidy is implemented. Despite its technical difficulty, a formal analysis when the resource is large can be pursued in future research efforts.

6 Appendix

6.1 Proof of Lemma 1

The expected revenue for contestant 1 and 2 are:

$$L_1 = \frac{e_1^r V}{e_1^r + e_2^r} - c_1 e_1;\ L_2 = \frac{e_2^r V}{e_1^r + e_2^r} - c_2 e_2. \tag{28}$$

Assume there exists a pure-strategy equilibrium where contestant 1 and 2 make efforts e_1^* and e_2^* respectively. First order conditions require

$$\frac{\partial L_1}{\partial e_1}|_{e_1=e_1^*, e_2=e_2^*} = 0; \frac{\partial L_2}{\partial e_2}|_{e_1=e_1^*, e_2=e_2^*} = 0.$$

which implies

$$e_1^{*r} e_2^{*r} rV = c_1 e_1^* (e_1^{*r} + e_2^{*r})^2;\ e_1^{*r} e_2^{*r} rV = c_2 e_2^* (e_1^{*r} + e_2^{*r})^2. \tag{29}$$

Solving (29), we have

$$e_1^* = \frac{c_1^r c_2^r rV}{c_1 (c_1^r + c_2^r)^2},\ e_2^* = \frac{c_1^r c_2^r rV}{c_2 (c_1^r + c_2^r)^2}. \tag{30}$$

To ensure the existence of a pure-strategy equilibrium, all contestants' participation constraints must hold, substituting (30) into (28), we derive:

$$L_1|_{e_1=e_1^*;\ e_2=e_2^*} = \frac{(1 - c^r(r-1))V}{(1+c^r)^2} \geq 0;$$

$$L_2|_{e_1=e_1^*;\ e_2=e_2^*} = \frac{c^r(1 + c^r - r)V}{(1+c^r)^2} \geq 0.$$

Thus, we write

$$L_1|_{e_1=e_1^*;\ e_2=e_2^*} \geq 0 \Leftrightarrow c^r \geq r-1; \tag{31}$$

$$L_2|_{e_1=e_1^*;\ e_2=e_2^*} \geq 0 \Leftrightarrow c^r(r-1) \leq 1. \tag{32}$$

When $r \leq 1$, we always have $c^r(r-1) \leq 1$ and $c^r \geq r-1$. So when $r \leq 1$, the equilibrium always exists. When $r > 1$, to make sure the equilibrium exists, by (31) and (32), the following condition must hold:

$$r-1 \leq c^r \leq \frac{1}{r-1}. \tag{33}$$

A necessary condition for (33) is $r \leq 2$, otherwise $r-1 > 1/(r-1)$. When $r \leq 2$, we always have $r-1 \leq c^r$. So we only need to focus on $c^r \leq 1/(r-1)$. When $r > 1$, we see that c^r increases in r and $1/(r-1)$ decreases in r. Thus, when $r > 1$, we need $r \leq \overline{r}$, where $\overline{r}$ satisfies $c^{\overline{r}} = 1/(\overline{r}-1)$. In summary, we can safely conclude that to ensure players' participation constraints hold, we need $0 \leq r \leq \overline{r}$, where $\overline{r}$ satisfies $c^{\overline{r}} = 1/(\overline{r}-1)$. It is simple to check that when c increases from 1 to $+\infty$, $\overline{r}$ will decreases from 2 to 1.

Now we want to show that the second order conditions hold at equilibrium when $e_1 = e_1^*$, $e_2 = e_2^*$. For player 1,

$$\frac{d^2L_1}{de_1^2}|_{e_1=e_1^*;\ e_2=e_2^*} = -\frac{e_2^{*r}e_1^{*-2+r}rV}{(e_1^{*r}+e_2^{*r})^3}[(r+1)e_1^{*r}-(r-1)e_2^{*r}]$$

Notice that (29) implies that $e_1^{*r} = (c_2/c_1)e_2^{*r} = ce_2^{*r}$,

$$\begin{aligned}\frac{d^2L_1}{de_1^2}|_{e_1=e_1^*;\ e_2=e_2^*} &= -\frac{e_2^{*r}rVe_1^{*-2+r}}{(e_1^{*r}+e_2^{*r})^3}[(r+1)(ce_2^*)^r-(r-1)e_2^{*r}] \\ &= -\frac{rVe_1^{*-2+r}e_2^{*2r}}{(e_1^{*r}+e_2^{*r})^3}[(r+1)c^r-(r-1)].\end{aligned}$$

As $[(r+1)c^r-(r-1)] > 1+r+1-r > 0$, we have

$$\frac{d^2L_2}{de_2^2}|_{e_1=e_1^*;\ e_2=e_2^*} < 0.$$

For player 2,

$$\frac{d^2L_2}{de_2^2}|_{e_1=e_1^*;\ e_2=e_2^*} = \frac{e_1^{*r}e_2^{*-2+2r}rV}{(e_1^{*r}+e_2^{*r})^3}[c^r(r-1))-(r+1)].$$

As (32) implies $c^r(r-1) \leq 1$, $c^r(r-1)) - (r+1) < 0$, we have

$$\frac{d^2L_2}{de_2^2}|_{e_1=e_1^*;\ e_2=e_2^*} < 0.$$

6.2 Proof of Proposition 1

From (11), we can derive that

$$\frac{dTE^S}{ds} >, < \frac{dTE^W}{ds}$$

if and only if $f(r) >, < 0$, where

$$\begin{aligned} f(r) &= [c + (1+c)r] - c^r[c(r-1) + r] - c^r[1 + (1+c)r] + [(r-1) + cr] \\ &= 2r(c+1) + (c-1) - c^r[2r(c+1) - (c-1)]. \end{aligned} \tag{34}$$

It is straightforward to see that $f(r) > 0$ when $2r(c+1) - (c-1) \leq 0$, i.e., $f(r) > 0$ when $r \leq (c-1)/2(c+1)$. It can also be proved that $f(r) < 0$ when $r \geq 1$. This is because at the point $r = 1$, $f(r) = 1 - c^2 < 0$ and

$$\frac{df(r)}{dr} = -\{2(1+c)(c^r - 1) + c^r(1 - c + 2(1+c)r)Log[c]\} < 0. \tag{35}$$

Therefore, we have shown that $f(r) > 0$ when $r \leq (c-1)/2(c+1)$ and $f(r) < 0$ when $r \geq 1$. The following question would be what happens when $(c-1)/2(c+1) < r < 1$. Using (34), we can show that $f = 2(c-1) > 0$ when $r = (c-1)/2(c+1)$ and $f < 0$ when $r = 1$. Using (35), we can derive that $df(r)/dr < 0$ when $(c-1)/2(c+1) < r < 1$, i.e., $f(r)$ is strictly decreasing when r increases. Thus, we conclude that there must exist a r^* where $f(r^*) = 0$ and $(c-1)/2(c+1) < r^* < 1$. It follows that when $r > r^*$, $f(r) < 0$; and when $r < r^*$, $f(r) > 0$.

Because of $f(c, r^*) = 0$, we derive

$$df = \frac{\partial f}{\partial c}dc + \frac{\partial f}{\partial r^*}dr^* = 0 \tag{36}$$

$$\Rightarrow \frac{dr^*}{dc} = -(\frac{\partial f}{\partial c} / \frac{\partial f}{\partial r^*}). \tag{37}$$

It is simple to derive that

$$\frac{\partial f}{\partial r^*} = -\{2(1+c)(c^{r^*} - 1) + c^{r^*}(1 - c + 2(1+c)r^*)Log[c]\} < 0 \tag{38}$$

and

$$\frac{\partial f}{\partial c} = 2r^* + 1 - c^{r^*-1}[2(1+c)r^{*2} + (1+c)r^* - c]. \tag{39}$$

Since $f(r^*) = 0$, from (34) we derive

$$c^{r^*} = \frac{2r^*(c+1) + (c-1)}{2r^*(c+1) - (c-1)} \tag{40}$$

Substituting (40) into (39), we have

$$\frac{\partial f}{\partial c} = -r^*\{\frac{4r^{*2}(c+1)^2 - (c-1)^2 - 8c}{c[(1-c)+2(1+c)r^*]}\}.$$

Since $(1-c)+2(1+c)r^* > 0$, we derive that $\partial f/\partial c < 0$ if and only if

$$4r^{*2}(c+1)^2 - (c-1)^2 - 8c > 0$$

$$\text{i.e., } r^* > \frac{\sqrt{8c+(c-1)^2}}{2(c+1)} = \widetilde{r}. \tag{41}$$

Substituting $\widetilde{r}$ into (34), we can show that $f(\widetilde{r}) > 0$ for all $c > 0$. It must be the case that $r^* > \widetilde{r}$ since $f(\widetilde{r}) > f(r^*) = 0$ and $df/dr < 0$. Thus, $\partial f/\partial c < 0$. Therefore, we derive

$$\frac{dr^*}{dc} = -(\frac{\partial f}{\partial c}/\frac{\partial f}{\partial r^*}) < 0.$$

Now we have shown that r^* decreases when c increases. By numerical analysis, we can derive that $r^* \approx 0.708$ when $c \to 1$ and $r^* \approx 0.5$ when $c \to +\infty$.[23]

6.3 Proof of Proposition 2

From (19), we can see that when $0 < r \leq 1$,

$$c - c^r[r - c(1-r)] = c - c^r r + c^{r+1}(1-r) > 0$$

since $c > c^r r$ and $c^{r+1}(1-r) > 0$. Thus, (19) > 1, i.e.,

$$\frac{dTE^S}{ds} > \frac{dTE^P}{ds} \text{ when } 0 < r \leq 1.$$

From (17), it is straightforward to see that when $c^r + 1 - (1+c)r > 0$,

$$\frac{dTE^W}{ds}/\frac{dTE^P}{ds} > 1.$$

Let $g = c^r + 1 - (1+c)r$, when $r = 1$, $g = 0$. We can guarantee that $g > 0$ for any $r > 1$ as long as

$$\frac{dg}{dr}|_{r=1} = c\log c - (1+c) \geq 0 \tag{42}$$

since

$$\frac{d^2g}{dr^2} = c^r(\log c)^2 > 0.$$

[23] Using Mathematica, we calculate that when $c = 1 \times 10^{-9}$, $r^* \approx 0.708$ and when $c = 1 \times 10^9$, $r^* \approx 0.5$.

Using (42), we derive that at point $r = 1$,

$$\frac{d^2 g}{drdc} = \log c > 0.$$

Thus, to ensure (42) holds, it requires that $c > \overline{c} \approx 3.6$, where $\overline{c}$ satisfies

$$\overline{c} \log \overline{c} - (1 + \overline{c}) = 0.$$

6.4 Proof of Lemma 2

a) In equilibrium, every player is making effort which must satisfy (22). By adding up (22) for each player, we write

$$\sum_{i=1}^{n} \frac{d\pi_i}{de_i} = \frac{\sum_{i=1}^{n}(\sum_{j \neq i} e_j)}{(e_i + \sum_{j \neq i} e_j)^2} V - \sum_{i=1}^{n} c_i = 0. \tag{43}$$

It is easy to see that

$$(e_i + \sum_{i=1}^{n} e_j)^2 = (\sum_{i=1}^{n} e_i)^2; \tag{44}$$

$$\sum_{i=1}^{n}(\sum_{j \neq i} e_j) = \sum_{i=1}^{n}\{(\sum_{i=1}^{n} e_i) - e_i\} = (n-1)\sum_{i=1}^{n} e_i. \tag{45}$$

Substituting (44) and (45) into (43),

$$\sum_{i=1}^{n} \frac{d\pi_i}{de_i} = \frac{V(n-1)}{\sum_{i=1}^{n} e_i} - \sum_{i=1}^{n} c_i = 0,$$

it follows that

$$TE = \sum_{i=1}^{n} e_i = \frac{V(n-1)}{\sum_{i=1}^{n} c_i}.$$

b) For any two players with $c_p < c_q$, by using (22) we can write:

$$\frac{d\pi_p}{de_p} = \frac{V\ (\sum_{i=1}^{n} e_i - e_p)}{(\sum_{i=1}^{n} e_i)^2} - c_p = 0; \tag{46}$$

$$\frac{d\pi_q}{de_q} = \frac{V\ (\sum_{i=1}^{n} e_i - e_q)}{(\sum_{i=1}^{n} e_i)^2} - c_q = 0. \tag{47}$$

Then, we further derive

$$\frac{(\sum_{i=1}^{n} e_i) - e_p}{(\sum_{i=1}^{n} e_i) - e_q} = \frac{c_p}{c_q} < 1. \tag{48}$$

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