“Tragedy of the Commons” *Revisited*

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Abstract

In this paper, I consider the regulation standard as a policy variable and investigate the effect of its choice when the law enforcement is costly and imperfect. If the enforcement probability depends only on the monitoring expenditures, the violation rate remains the same for any quantity regulation stricter than the marginal regulation. If the enforcement probability depends on the violation rate as well as the expenditures, I obtain an even more surprising result that the violation rate and the traffic volume can be both decreased as the regulation gets less strict. Intuitively, a less strict regulation makes the detection probability higher because the regulatory authority only needs to regulate less vehicles, which makes drivers less likely to violate the regulation.

Key Words: congestion, law enforcement, marginal regulation, regulation standard, tragedy of the commons

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1 Introduction

Some economic activities create serious negative externalities to others. Those activities are legally prohibited by being classified as "crimes." However, some other activities which are not socially undesirable *per se* may generate a serious social concern if the number of those who engage in the activities is large. This problem which is referred to as congestion has been dramatically portrayed in "Tragedy of the Commons" by Hardin (1968). The congestion problem also provides a justification for the government's intervention just as crimes.

If people are selfish so that a larger than the socially desirable number of people engage in the activities, the government may restrict the number of activities by the direct quantity regulation. Although this seems to be an easy solution to the congestion problem, the effectiveness of the regulation policy is not so transparent if the enforcement is not perfect.

In this paper, I consider the regulation standard as a policy variable and investigate the effect of its choice when the law enforcement is imperfect. The government's enforcement efforts cannot detect all the violations but only some proportion of them probabilistically. Also, the probability that a violator is caught is proportional to the enforcement efforts. However, more realistically speaking, the probability does not depend on the enforcement expenditures but also on the number of violators. For instance, if all of the population violates the regulation, it is almost infeasible to catch them all, implying that a violator is very unlikely to be caught. On the other hand, if only one of the population commits a violation, he will be most likely to be caught. Thus, I will consider two enforcement probabilities, the (usual) one that depends only on the expenditures[1] and the other that depends on the violation rate as well as on the expenditures.

The main issue that will be addressed in this paper is whether it alleviates the congestion problem to impose a stricter quantity regulation. I show that a stricter quantity regulation does not necessarily help alleviating the congestion problem in both cases. To convey the insight clearly, consider the case of driving. In a decentralized environment, an equilibrium divides the population into two groups, driving group and the non-driving group, and the

[1] Most of the existing literature on law enforcement adopts this simple form of enforcement probability. See Polinsky and Shavell (1979, 1984, 1991) and Malik (1990) etc.

driving group consists of people whose valuations from driving exceed some cutoff value, while the non-driving group consists of those whose valuations are lower than the value. In the first case of the expenditure-determined probability, let me call the person whose valuation from driving is equal to the cost of violating (driving) *marginal individual* and name permitting driving only to those whose valuation is higher than or equal to the marginal individual's valuation *marginal regulation*. Then, if the quantity regulation is made stricter than the marginal regulation, those who get forbidden to drive under the stricter regulation have a higher valuation of driving than those who were forbidden under a less strict standard, while their cost of illegal driving is the same as the latter. So, they will keep driving thereby violating the regulation under the stricter regulation. This implies that the traffic volume remains the same as under a less strict regulation. This leads to the neutrality result that the violation rate is independent on the regulation standard insofar as the regulation is stricter than the marginal regulation. In the second case, I obtain an even more surprising result that as the quantity regulation becomes stricter, the violation rate and the traffic volume are both increased, thus aggravating the congestion. Intuitively, a stricter standard increases the number of violations thereby making it more difficult to detect a violation which again yields a higher violation rate.

The paper is organized as follows. In Section 2, I set up the model without regulation and compare the Nash outcome with the social optimum. The usual congestion outcome is reconfirmed. In Section 3, I introduce the policy of direct quantity regulation to alleviate the congestion and examine the effect of a harsher regulation policy. Section 4 contains the conclusion.

2 Basic Model without Regulation

There are a continuum of drivers. Each driver decides to drive a car or not. Driving a car increases congestion on a road and slows down traffic.

To be more specific, define the following notation.

v = benefit from driving a car

$g(v)$ = probability density of v ($g > 0$ for all $v \geq 0$)

$x =$ rate of traffic volume ($x \in [0,1]$)
$c(x) =$ delay cost due to congestion ($c' > 0$, $c'' > 0$ and $c \geq 0$ for all $x \in [0,1]$)

First, consider the private optimum. Given that the mass of those who drive is x^N, a driver drives a car iff $v \geq c(x^N)$. Thus, a driver will drive for any $v \geq v^N$ such that

$$v^N = c(x^N), \tag{1}$$

where $x^N = \int_{v^N}^{\infty} g(v)dv$.

Now, consider the social optimum. Noting it is socially desirable that only the highest valuation drivers of mass x drive given any mass x, let the socially optimal traffic volume be x^S and the corresponding infimum valuation of those who drive be v^S. Then, the social optimum, (v^S, x^S), is determined by maximizing the total social value net of the total delay cost, i.e.,

$$\max_{\hat{v}} V = \int_{\hat{v}}^{\infty} (v - c(x))g(v)dv, \tag{2}$$

where $x = 1 - G(\hat{v})$.

By using the relation $x = 1 - G(\hat{v})$, one can write the first order condition as

$$\frac{\partial V}{\partial \hat{v}} = \left[c(x^S) - v^S + c'(x^S) \int_{\hat{v}}^{\infty} g(v)dv \right] g(v^S) = 0. \tag{3}$$

Equation (3) has the usual interpretation that the marginal social benefit from an increase in traffic equals the marginal social cost due to more congestion. The third term in the square bracket is the negative externality that an additional traffic incurs. Thus, $c^S(x) \equiv c(x) + c'(x) \int_{\hat{v}}^{\infty} g(v)dv$ is the social cost associated with the traffic.

Let us compare x^S and x^N. It is easy to see that $x^N > x^S$ by comparing equation (1) and (3). Since $c'(x) > 0$ for all x, equation (3) simply implies $v^S > c(x^S)$, which in turn implies that $x^S < x^N$; hence, too much driving (congestion). (See Figure 1.) This is simply a continuum version of Tragedy of the Commons.

3 Regulation

The government may want to alleviate the congestion problem by the direct control of quantity. The problem with the quantity regulation is that it is difficult to tell who is to be penalized when the congestion exceeding the regulated quantity actually occurs. A theoretical solution is to give priority to individuals with higher valuation, but their individual valuations are generally *ex ante* unobservable in reality. Nonetheless, I assume that each consumer's valuation is observable and verifiable so that the government can permit driving in the order of high valuations. This assumption can be justified as follows. First, the government may well tell each individual's valuation *ex post*, that is, by inspection after it caught a driver violating the regulation. Second, the government may obtain an imperfect signal for each individual's valuation by the appearance of vehicles. For example, in some countries, driving only large vehicles on bus (car pool) lanes is permitted.

For the analysis of the regulation, I need the following notation.

$\bar{x}$ = regulation level (quantity to be permitted)
p = probability that a violator is detected and caught
f = fine
$w(x)$ = total cost of a violator

By definition of $w(x)$, it directly follows that $w(x) = c(x) + pf$. Regarding the detection probability, I consider two cases; (i) $p = p(e)$ and (ii) $p = p(e, r)$ where e is the monitoring expenditures and r is the overall violation rate. Throughout the paper, I assume that e and f are fixed so as to focus on the effect of the regulated quantity as the only policy variable.[2]

3.1 When the detection probability depends only on expenditures

To concentrate on the complementary role of the quantity to be regulated($\bar{x}$) as a means to alleviating congestion, suppose that e and f are so low that $v(x^S) > w(x^S)$, that is, $p(e)f < v(x^S) - c(x^S)$ where $v(x)$ is the lowest valuation in the traffic x. (See Figure 2.)

With regulation, individuals prohibited from driving decides whether to violate or not by comparing his valuation from the violation v with his cost incurred $w(x)$. First, consider $\bar{x} = x_0$ where $x^S < x_0 < x^N$. It is easy to show that the Nash equilibrium traffic is $x^* = \bar{x}$, implying that an individual with v drives if and only if $v \geq v_0$ where $x_0 = 1 - G(v_0)$. To see

[2]If e and f are choice variables, there exists the unique value for $p(e)f$ to implement the social optimum x^S, and then, by the well-known argument of Becker (1968), f is determined at the maximum level possible.

this, an individual with $v < v_0$ finds that his valuation is less than his cost when he deviates by driving, since $v < v_0 = w(x_0)$. In this equilibrium, no one violates the regulation. Now, consider $\bar{x} = x_1$ where $x^S < x_1 < x_0$. It is also easy to show that the equilibrium traffic volume is still x_0. Let me show why the traffic of $x_0 - x_1$ violates the regulation. Let $v_1 = 1 - G(x_1)$ and take any $v \in [v_1, v_0)$. If he drives by violation, his valuation exceeds his cost, since $v > v_0 = w(x_0)$. Thus, he violates the regulation. Since any regulation $\bar{x}$ less stricter than x_0 has the same effect on congestion, I will call v_0 and x_0 marginal individual and marginal quantity, and call the regulation with $\bar{x} = x_0$ marginal regulation. A change in the actual traffic volume with respect to a change in the quantity to be regulated is drawn in Figure 3. The discussion so far leads to the following proposition.

Proposition 1 *Suppose that the detection probability depends only the expenditures and, the fine and monitoring expenditures are fixed to a level such that $p(e)f < v(x^S) - c(x^S)$. Then, the social optimum cannot be implemented for any quantity regulation $\bar{x}$. Moreover, for any quantity regulation stricter than the marginal regulation ($\bar{x} \leq x_0$), the actual traffic amount is the same, and some violations occur except the case of marginal regulation.*

This proposition suggests that regulating the traffic more strictly than the marginal quantity x_0 is unnecessary in the sense that it does not improve the congestion problem at all. Thus, this implies that if the regulation-violating behavior itself is considered as socially undesirable,[3] the marginal regulation $\bar{x} = x_0$ is most preferred. The intuition is as follows. By definition, the marginal individual's valuation v_0 is equal to the total cost from violation. The individual who is forbidden to drive by a stricter regulation has the valuation higher than v_0 which is equal to the total cost from violation. Thus, he still has an incentive to drive.

3.2 When the detection probability depends on the violation rate as well

In this section, I consider the detection probability that depends on the overall violation rate as well as the expenditures. Thus, I assume that $p = p(e, r)$ with $p_1 \equiv \partial p / \partial e > 0$ and

[3] One possible concern about this behavior is that it could increase the overall propensity to violate regulation in the long run.

$p_2 \equiv \partial p/\partial r < 0$. The assumption that $p(e,0)f < v(x^S) - c(x^S)$ will be preserved.

Unlike in the case that $p = p(e)$, the total cost of violation $w(x)$ depends on the quantity regulation $\bar{x}$ in this case, since $p(e,r) = p(e, x - \bar{x})$. So, I will use the notation $w(x \mid \bar{x})$ to express $w(x)$ given the quantity regulation $\bar{x}$. In fact, the shape of $w(x \mid \bar{x})$ is crucial to the analysis of this case.

Let me illustrate the curve $w(x)$ for two regulation quantities, $\bar{x} = x^S$, x' in Figure 4. If $\bar{x} = x^S$, the equilibrium traffic is x^* and the corresponding violation rate can be measured by $x^* - x^S$. If the quantity to be permitted is increased to $\bar{x} = x'$, the equilibrium traffic is x^{**} and the violation rate is $x^{**} - x'$. Hence, by a less harsh regulation, the equilibrium traffic is decreased and so is the violation, thereby congestion is alleviated. Such a traffic decrease occurs continuously up to the tangent point of the two curves $v = 1 - G(x)$ and $w(x) = c(x) + p(e,r)f$. The tangent point and the corresponding traffic volume are illustrated as E and x^{***} in Figure 4. Let the regulation quantity be $\bar{x}^*$. Then, the violations under this regulated quantity can be measured by $x^{***} - \bar{x}^*$. Now, we can go further. If the regulation is made even less strict, say $\bar{x} = \bar{x}^* + \epsilon$ and above, one can show that the traffic volume is discontinuously decreased and they go up smoothly along the regulation quantity. Thus, the optimal regulation is $\bar{x} = \bar{x}^* + \epsilon$. Note that the optimal regulation quantity cannot be x^S as long as $v(x^S) > w(x^S \mid x^S)$.

Proposition 2 *Suppose that the detection probability depends on the offense rate as well as the expenditures and the fine and monitoring expenditures are fixed to a level such that $p(e,0)f < v(x^S) - c(x^S)$. Then, the social optimum cannot be implemented for any quantity regulation. Also, the optimum regulation minimizing the traffic volume occurs when $\bar{x} = \bar{x}^* + \epsilon$. As the regulation quantity $\bar{x}$ becomes stricter, the traffic volume decreases along the line $x = \bar{x}$ for $\bar{x} > \bar{x}^*$ and jumps up discontinuously at $\bar{x} = \bar{x}$, and then keeps increasing for $\bar{x} < \bar{x}^*$.*

Proof. See the appendix.

Figure 5 shows the change in the actual traffic volume with a change in the quantity regulation. This proposition suggests that the effect of a stricter regulation on the traffic volume is not monotonic. As the regulation gets stricter, i.e., $\bar{x}$ becomes smaller and smaller from the optimum regulation $\bar{x}^*$, the traffic volume is increased. Intuitively, as the regulation is stricter, more vehicles are classified as violating for the same x which implies a lower detection probability. Since this makes the cost of violation lower in terms of expected value, the number of actual violations is increased. If the detection probability approaches zero, the traffic volume will converge to x^N since $w(x)$ will become very close to $c(x)$. However, as the regulation gets less strict for $\bar{x} > \bar{x}^*$, the cost of violation exceeds its cost regardless of the number of violators, implying no one violates the regulation. Thus, the traffic volume should be equal to the regulated quantity, which means that the traffic volume increases as the regulation becomes less harsh. The economic insight behind the optimal regulation in this case is as follows. If the regulated quantity is high enough, the detection probability is high due to the small number of violators, so no one will violate. Then, the optimal regulation is to choose the most harsh quantity among those quantities that induce no violation.

4 Conclusion

Congestion has been serious social concerns in many sectors involving common properties, for instance, transportation on the road, communication over the Internet, fishery in the ocean etc. Lots of regulation policies to alleviate the congestion problem have also been proposed. One often observed regulation in driving is to forbid vehicles with some plate numbers (e.g., 1 or 2 ending numbers on Monday, 3 or 3 ending numbers on Tuesday ...) to drive. The regulation is sometimes made stricter by forbidding vehicles with odd/even numbers when the congestion is quite severe. However, this paper suggests that such a policy may aggravate the congestion problem unless it is accompanied by higher monitoring expenditures.

Appendix

Proof of Proposition 2: From the discussions in the text, it suffices to show that for all $\bar{x} > \bar{x}^*$, the actual traffic volume is $x = \bar{x}$. It is clear that all individuals with $v \geq v(\bar{x})$ drive cars. For any individual with $v < v(\bar{x})$, his cost of violation exceeds his benefit of driving since $w(x) > v(x)$ for all $x > \bar{x}$. This implies that he will have no incentive to deviate by violating (driving). Hence, the resulting equilibrium traffic volume is $x = \bar{x}$.

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